UC-NRLF
$C 189 430

LIBRARY
OF THE
UNIVERSITY OF CALIFORNIA.
GIFT OF
U.S. Weather bureau
Class

Treat as monograph.
What class?

W. B. No. 316.

U. S. DEPARTMENT OF AGRICULTURE.

WEATHER BUREAU.

STUDIES ON THE CIRCULATION OF THE ATMOSPHERES OF THE SUN AND OF THE EARTH.

Reprints from the Monthly Weather Review, October and November, 1903, and January, February, April, May, and June, 1904.

LIBRARY OF UNIVERSITY OF CALIFORNIA

BY

FRANK H. BIGELOW, M. A., L. H. D.,

PROFESSOR OF METEOROLOGY.

Prepared under the direction of WILLIS L. MOORE, Chief U. S. Weather Bureau.

LIBRARY OF THE UNIVERSITY

WASHINGTON:
WEATHER BUREAU.
1904.

W. B. No. 316.

U. S. DEPARTMENT OF AGRICULTURE.
WEATHER BUREAU.

STUDIES ON THE CIRCULATION OF THE ATMOSPHERES OF THE SUN AND OF THE EARTH.

Reprints from the Monthly Weather Review, October and November, 1903, and January, February, April, May, and June, 1904.

BY

FRANK H. BIGELOW, M. A., L. H. D.,
PROFESSOR OF METEOROLOGY.

Prepared under the direction of WILLIS L. MOORE, Chief U. S. Weather Bureau.

LIBRARY OF THE UNIVERSITY OF CALIFORNIA

WASHINGTON:
WEATHER BUREAU.
1904.

QC931
B5

Gift of
U.S. Weather Bureau

CONTENTS.

128549

LIBRARY OF THE UNIVERSITY OF CALIFORNIA

STUDIES ON THE CIRCULATION OF THE ATMOSPHERES OF THE SUN AND OF THE EARTH.

I.—THE CIRCULATION OF THE SUN'S ATMOSPHERE.

HISTORICAL REVIEW.

That the solar atmosphere is circulating in accordance with the laws governing the convective and radiative action of a large mass of matter contracting by its own gravitation, is so evident that numerous efforts have been made to determine what these laws are, or at least to discover some reliable clue to a beginning of scientific research in that direction. The application by R. Emden[1] of H. von Helmholtz's method of adapting the general equations of motion to a solar mass, appeared to be a step in the right direction; further attention was called to the possibilities of this solution in my Report on Eclipse Meteorology,[2] pages 71–74. In June, 1902, Sir Norman Lockyer and Dr. W. J. S. Lockyer[3] published their suggestive curve of the percentage frequency of the solar prominences derived from the Italian observations for each 10° of solar latitude north and south of the equator. This curve interested me because it appeared to identify the distinctly solar phenomena with the short period curves which I had worked out in the terrestrial magnetic field and in the meteorological field of the United States, and first published in December, 1894,[4] afterwards republishing them in 1898.[5] A study of the difficult subject of inversion of periodic effects in magnetic and meteorological phenomena discovered at that time has been actively pursued by the Weather Bureau for the past ten years, and evidence is being accumulated, not only here but by others, of the existence and importance of the fact of inversion in the magnetic phenomena, the pressures, and the temperatures of the earth generally. The solar prominence curve suggested also the possibility of obtaining more decisive evidence of solar and terrestrial synchronisms than that afforded by the solar-spot frequency curve (which is apparently only a sluggish register of the true solar output of energy), because the terrestrial magnetic field and the meteorological elements show minor variations that are only feebly indicated in the solar-spot curve. The prominence frequency curves brought out distinctly for the sun the minor fluctuations that had been already found in the earth's atmosphere.

My first computations on the amplitudes of the deflecting forces which disturb the normal terrestrial magnetic field were computed for the years 1878–1893, using the records of several European magnetic stations. To have extended the same computation to the years 1841–1900, inclusive, would have required a vast amount of labor; as an equivalent, the deflections of the horizontal force alone, without the declination and vertical components, were derived by the construction of a series of graphical curves covering these sixty years, from which the mean ordinates were computed. The result was shown in my paper on Cosmical Meteorology, July, 1902.[6] The same variation curve was found from the horizontal force for the years 1878–1893 as that previously given by the computed σ curve, and it was therefore proper to conclude that this extension of the original computation in both directions was sufficiently correct for the purpose of the discussion. Furthermore, the prominence frequencies presented the material for studying the solar activity by zones, and the result of my compilation to determine the law of the movement of the points of prominence maxima in latitude was read before the American Association for the Advancement of Science on December 28, 1902, and published in the MONTHLY WEATHER REVIEW, January, 1903.[7] I there showed that in each hemisphere the maxima of prominence frequency are grouped in two zones, and that in the zones near the equator, in latitudes about 20°, the maxima of frequency approach that plane in common with the sun spots and faculæ during the 11-year period, while in the zones in latitudes 50°–70°, the maxima simultaneously move toward the poles. This indicates a characteristic tendency of the solar circulation to spread from the middle latitudes toward the equator and toward the poles in two independent branches. In a paper[8] published in March, 1903, the Lockyers obtained a similar result for the same phenomena. They gave the life history of the sun in the separate 11-year periods between 1872–1901, whereas my paper had grouped these three available periods together for the sake of finding the average law. Dr. A. Ricco[9] has published similar studies of the movements of prominences in latitude for the years 1880–1902. The subject of the average distribution of the solar spots in longitude on the sun has been discussed by Dr. A. Wolfer,[10] and from it he derived some determinations of the solar rotation in different latitudes. In my paper of January, 1903, I stated that besides a study of the variable distribution of the prominences in latitude, an effort was being made by me to discover some clue as to their distribution in longitude, in order to learn whether or not there was an accumulation on certain meridians, and it is the result of this work that is contained in the present paper. We have discovered an unexpectedly clear insight into the solar circulation, and this tends to strengthen the line of argument which I have been developing during the past fifteen years to explain the

[1] Eine Beobachtung uber Luftwogen. R. Emden. Wied. Ann. LXII, p. 62, 1897, and Astrophysical Journal, January, 1902.

[2] Eclipse Meteorology and Allied Problems. Frank H. Bigelow. Weather Bureau Bulletin I. 1902.

[3] On some Phenomena which suggest a short Period of Solar and Meteorological Changes. By Sir Norman Lockyer, K. C. B., F. R. S., and William J. S. Lockyer, M. A., Ph. D., F. R. A. S. Received June 14. Read June 19, 1902. Addendum. Dated June 26. Proc. Roy. Soc. Vol. 70.

[4] Inversion of Temperatures in the 26.68 Day Solar Magnetic Period. Frank H. Bigelow. Am. Jour. Sci. Vol. XLVIII, December, 1894.

[5] Report on Solar and Terrestrial Magnetism in their Relations to Meteorology. Frank H. Bigelow. Weather Bureau Bulletin No. 21. 1898.

[6] A Contribution to Cosmical Meteorology. Monthly Weather Review, July, 1902, Vol. XXX, p. 347.

[7] Synchronous Changes in the Solar and Terrestrial Atmosphere. Monthly Weather Review, January, 1903, Vol. XXXI, p. 9.

[8] Solar Prominence and Spot Circulation, 1872–1901. By Sir Norman Lockyer, K. C. B., F. R. S., and William J. S. Lockyer, Chief Assistant, Solar Physics Observatory, M. A. (Camb.), Ph. D. (Gott) F. R. A. S. Received March 17. Read March 26, 1903. Proc. Roy. Soc. Vol. 71.

[9] Le protuberanze solari nello'ultimo periodo undecennale. Mem. Spett. Ital., Vol. XXXII, 1903. A. Ricco.

[10] Publikationen der Sternwarte des Eidg. Polytech. Inst., Zurich. A. Wolfer. Bd. I, II, III, 1897, 1899, 1902.

mysterious synchronism at the earth, of which numerous symptoms have been noted, in many kinds of observations.

COMPILATION OF THE PROMINENCE OBSERVATIONS.

The prominences which appear on the edge of the disk of the sun have been carefully delineated by the Italian observers Secchi and Tacchini with stations at Rome and Palermo, also Ricco and Mascari, at Catania, working in cooperation, from March, 1871, till the present time in an unbroken series. Students of solar physics can not too gratefully acknowledge the value of the patient, laborious work which has been done by these observers, and the practical study of these data is likely to open up new and important lines of research. Beginning with March 1, 1871, the images of the solar disk have been published in the Memorie della Società degli Spettroscopisti Italiani, and they cover the time to the end of the century, except for a long gap from September, 1877, to January, 1884. I am informed by Dr. Ricco that the drawings for these missing years are in the archives of the Catania Observatory, and it is obvious that steps should be taken as soon as practicable to complete the published record, because the demand for the data is sure to increase, as can be inferred from the results indicated in this paper. On those graphical tables certain lines were drawn showing the position of the north and south poles and the equator of the sun, so that the disk could be readily divided into zones, passing first along the eastern limb from north to south, and then along the western limb from south to north.

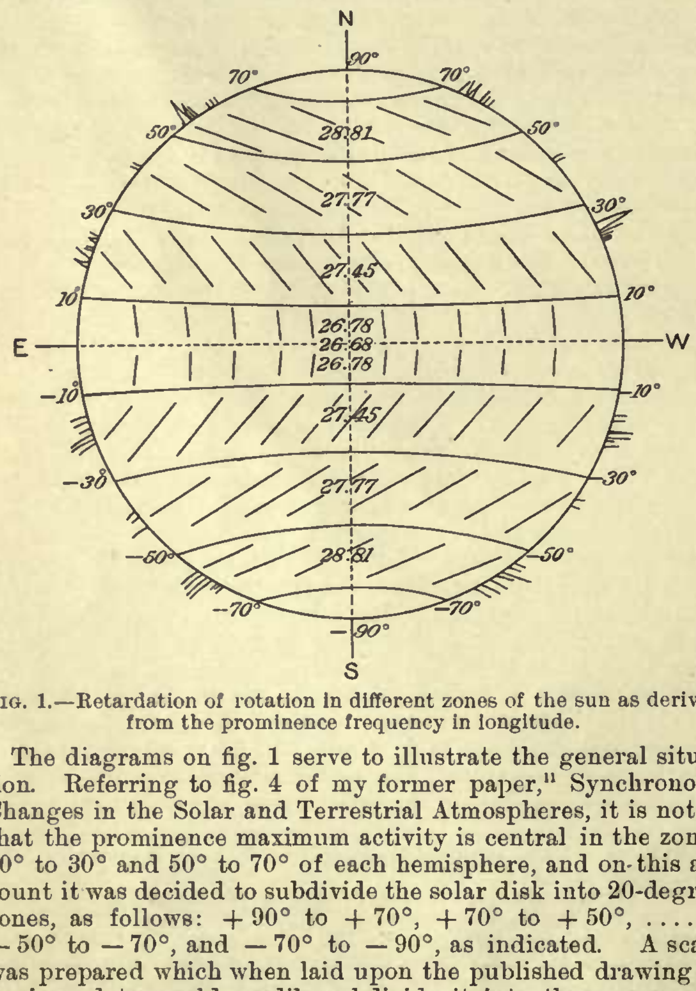


Fig. 1.—Retardation of rotation in different zones of the sun as derived from the prominence frequency in longitude.

The diagrams on fig. 1 serve to illustrate the general situation. Referring to fig. 4 of my former paper,[11] Synchronous Changes in the Solar and Terrestrial Atmospheres, it is noted that the prominence maximum activity is central in the zones 10° to 30° and 50° to 70° of each hemisphere, and on this account it was decided to subdivide the solar disk into 20-degree zones, as follows: $+90°$ to $+70°$, $+70°$ to $+50°$, ... $-50°$ to $-70°$, and $-70°$ to $-90°$, as indicated. A scale was prepared which when laid upon the published drawing of a given date would readily subdivide it into these zones on each side of the sun's limb.

For the sake of recording the relative energy of the solar output as registered in the prominences, a scale of estimation was adopted, as follows:

0 = an undisturbed limb for the zone.

1 = a minor disturbance.

2 = a somewhat extensive disturbance.

3 = a disturbance pronounced in altitude or along a considerable extent of the zone.

4 = a very large, emphatic agitation of the limb.

5 = the largest prominences, occurring but rarely.

The state of the limb was thus expressed in numbers of relative energy by estimation, care being exercised to make a similar relative number do duty whenever the style of the drawing changed from one draftsman to another. The computation sheets were arranged to allow the data for each of the nine zones to be collected together by years for the first compilation. For the second compilation the data belonging to the same zone for the successive years were brought together. Hence, the work of tabulating the data was repeated twice throughout the series. For an ephemeris I used the one already constructed from my computation on the variations of the terrestrial magnetic field, having the period 26.679 days and epoch June 13.72, 1887, as given on page 120, Bulletin No. 21, Solar and Terrestrial Magnetism. This is known to coincide very closely with the period of the *solar rotation at the equator*, and as it was one purpose of this research to test practically the working of this period, it was laid at the basis of the compilation. It makes no difference what ephemeris and period are adopted, since any periodic phenomenon not falling upon that period will show a gradual departure from it by the trailing of the numbers on the sheet from left to right, if the period is too short, or from right to left, if it is too long.

An example of the use of the ephemeris and the result is given in Table 1. One point should be especially noted in this connection, and that is as follows: *The same meridian of the sun is seen twice in a single rotation, first as the eastern limb, and second, thirteen days later, as the western limb.* Whatever may be the intrinsic activity of the sun at a given zone and on a given meridian, that display becomes visible twice, first to the east and second to the west. During the passage of that meridian across the sun's disk the record is wanting so far as this series is concerned, though it could of course be studied otherwise by means of the spectro-heliographic photographs. Thus, as the successive meridians come to the edge of the disk, their output is recorded on the respective drawings. When these are collated with the equatorial period, whatever characteristics they may have which would imply special centers of solar activity will gradually emerge upon the numerical tables. As it is not possible to reproduce these extensive tables in this connection, two specimens of the second collection are shown on Table 1 for the years 1891 and 1892 in succession, and for the zones $+50°$ to $+70°$ and $+10°$ to $+30°$. Imagine that similar tables for zones $+50°$ to $+70°$ extend from 1871 to 1900, inclusive, except for the gap from 1878–1883, arranged continuously so that the prominence concentration and depletion flows without break on the sheet from year to year. This process is extended to the 9 zones, each 20° in width. In the first collection of the data the highest number was 5, and this was very rarely entered. Since the same area on the sun is seen twice, there may be two entries within the same tabular area on the *first set of sheets*. In the *second set of sheets* these numbers are added together and entered as one, so that occasionally the figures 6, 7, 8 occur, as in Table 1. They represent the largest disturbance occurring in one small area of the sun, as defined by the latitude and longitude thus prescribed. If now the maxima show a tendency to trail across the sheet as indicated by the continuous lines drawn athwart the table, instead of being scattered at random, then this is evidence that the center of eruption itself rotates about the sun at a different rate from that laid down in the assumed

[11] Monthly Weather Review, January, 1903, Vol. XXXI, p. 17.

TABLE 1.—*The prominence energy in zones as collected on the 26.68-day period, showing retardation in different latitudes.*

Period 26.679 days; Epoch June 13.72, 1887.

Zone + 50° to + 70°.

			1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	
1891	Jan. 11	1		3	2												1	1												
	Feb. 6	2	1			1	2	2	2		4	2									2	4	4	2	3	2	2	2		
	Mch. 5	3		4	3		2			1															2					
	Apr. 1	4	2		2		4	3	2					1		1		1		2	2						1			
	Apr. 27	5						3	1										2		1									
	May 24	6	2		1				1	4	3	5	3	2	1				1			1		1						
	June 20	7	1			1				5	2	3	3	6	3		1	5	1	1	2	3								
	July 11	8	1							2	3	2	2	1	2	3	1	1	2	1	1	1	1	1			2		2	
	Aug. 12	9						1	1					2	4	8	5	4	5	4	5	4	4	3	1		1		1	
	Sep 8	10			1	2	6	7	8	6	4	6	4	1	1	2	5	5	3	7	7	7	6	3	4	3	3	3		
	Oct. 4	11	4	2	3		4	3	4	3	1		3	4	4	8	3	6	3	4	6	7	8	3	3	7	4	2	1	
	Oct. 31	12		4						5		4	2		1	2	3	3	3	3	3	4	4			3		4		
	Nov. 27	13	2	1	2			2	2	4	4	4	3	4	3	1					2	2	2	1	2	2	2			
	Dec. 23	14			1	2					2	1				1	1				2	6			1					16
1892.	Jan. 19	1		1	2	1				1				2			2	1	5	3	1	1	1					1	1	
	Feb. 15	2			4			1	2			3		1				4			1	2			3			2		
	Mch. 12	3	3	3	3		2	1	2	3	2	3	4			2	2	2			1	1	1	2	2	1	2	4		
	Apr. 8	4	6	6	3	2		4	3	3			1	1	1	4	8	1	1		3	2	3		1	6	3	5	2	
	May 5	5	3	1	2	3	3		2	2	4	3	3	3	2				3	3	1	1		2			2	3		
	May 31	6	1	2		2	2	3	3	3	2	4	6	5	2	4	1		1	4	3	3	3	2	1	1	1	1	2	
	June 27	7	4	3	2	1		2	2	3	3	3	3	2	3	7			1		3	2	5	6	5	2	4	2	1	
	July 24	8	3	1	1				3	2	1	5	3	4	2	3	4	4	6	3	4	3		2	3	2	3	2		
	Aug. 19	9		3	3	1		1		1	2	2	4	1	5	5	5	3	4	4	4	4	4		2	2	2	3	1	
	Sep. 15	10	4			3		3	1		2		1	1	1	5	3	4	5	1	5	4	1	1	1			1		
	Oct. 12	11	3	3		1	1	5	4		2		2	1	2	1	2		3	5	2	3	3	5	1	2	2	6		
	Nov. 7	12	6	3		1	2							3	1		2		2	3	6	3	3	4	5		6	4	3	
	Dec. 4	13		3	3	3			4		2						2	2	2		2				2					17

Zone +10° to +30°.

			1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	
1891	Jan 11	1			1	2									2	5	3	2	2				2				1	1		
	Feb. 6	2	2	5	2	1		1	1	1			1	1	2	2	1	2	1	1		2	1	1	1	4	3	1		
	Mch 5	3	2	3				1			1			2	1		6	6	2	6	3						2	2		
	Apr. 1	4	1		2	1	3	2	4	3				1	1	2	1	1	1	4	2	2						1		
	Apr. 27	5	1		1	1		3		1	3	2			1	3		1		1	3	1	3	1	1		1	3	1	
	May 24	6	1		1	2	2	1	1	4		1		1	5	8	2	1	2		3	4	4	3	4	1	4	5	2	
	June 20	7	3	3	3	2	1		1	3	4	3		7	3	7	3		1	2			3	2	3	2	3	3		
	July 11	8	3	5	1	2	1	3	2	6	4	4	2	1	3	4	5	5	1							2	4			
	Aug. 12	9	4	4	6	4	3	3	4	2	1	2	2	2	1	5	3	4			1	1			1		2	5	3	
	Sep 8	10	4	3	5	4	5	4	5	2	2	2	1	3			3	2		2		1				4	5			
	Oct. 4	11		2	4	1	3	1	2		2	1	2	1			1		1	3			1		1	1			1	
	Oct 31	12		2			1	2	3	2		3					4	1	2			1			4					
	Nov. 27	13					1	7		4	3	2		1	1	2	4	2					4	4	2	1	2	1		
	Dec. 23	14			1				2		2	2		2	1	1			1		2	6		3		1	1	1		14
1892	Jan 19	1	1					2	2	2			3	2	2	4	1			2	4		4			2			1	
	Feb 15	2			2	1			2			7	5	4	4		4	1	2		4	3			3	3				
	Mch 12	3	2	1		3	3			1		3	3	3	2	2					5	5	2	2	1					
	Apr. 8	4	1		2	6	5	3	2			1	4	3	1					1	2		4		4		3		1	
	May 5	5	2		2		1			3					1			2	1			3	1	1	6	1	3	5		
	May 31	6	5			1	3	2		2		3	4	3	2	3	4	2	3			2	3	2		3	2	1	1	
	June 27	7	3	6	5	1	4	1	6		1	2	4	3	4	5	3	2	4	1	1	2	2	3	3	5	2	3	1	
	July 24	8	1	4	2		1	3	4	2	1	4	2			3	4	1	4	4	3	2	1	5	5	6	3			
	Aug. 19	9	4	4	3	3	2	1	3	1	2	3			2	3	4	2	3	6	5	2	1	1		6	1	4		
	Sep. 15	10	2	2	3	3			3	3		1	1		1	4	1	3	5	4	7	3	2	5	3		2	2	1	
	Oct 12	11	3	2	1		2	1		2			1			3	3	1	3	3	4	3	2	2	2	2	2	3		
	Nov. 7	12	3			1			3	3		1	4	2	1		1	1		4	6	4	5	6	5			1		
	Dec. 4	13	2	4	5	5	2	1	2	1		1	2	2	1		2	3	1			2	2		3				2	
										16															15					

ephemeris. From such trails the angular retardation in different zones can be computed with considerable exactness. The reader will not receive a satisfactory impression of the distinctness with which this trailing at different rates in the several zones occurs, without an inspection of the entire series of tables, and it is hoped that they will be published in a special report, as the subject matter is evidently very important and suggestive for the solution of the fundamental problem of the mode of the internal solar circulation.

An examination of these sheets indicates that there is a marked tendency for the numbers to bunch themselves together in a very special manner. Between the successive years there is generally a depletion corresponding with the winter months, while the summer months are relatively full and complete. As pointed out in my paper on Synchronous Changes, this is evidently due to the fact that the relatively cloudy weather in Italy during the winter months made it impossible to secure so many days of observation as during the summer, and I conclude that the apparent concentration of the tables in the summer season is a meteorological effect, and should be treated as such in interpreting the results. At the same time there is a very similar concentration of the numbers along the days of the period, corresponding with a solar rotation, which can not be explained in that way, since it occurs as prominently in summer as in winter. It must apparently be referred back to some solar activity producing prominences on the two opposite sides of the sun. The *maximum numbers* not only trail downwards and to the right on the tables, but the *lines of maximum* also drift across the tables to the left, thus indicating retardation in the higher latitudes relative to the adopted equatorial period.

It may be mentioned in passing that this increase of activity of the sun on two opposite sides of its mass, as if a certain diameter had greater energy than the one at right angles to it, has already been detected by me in the meteorological field of the earth's atmosphere, and also in the terrestrial magnetic field, as shown on pages 91 and 92 of my Eclipse Meteorology and Allied Problems, and elsewhere. This persistent excess of outflowing energy on two opposite sides of the sun suggests the possibility that *the sun should be regarded as an incipient binary star*,[12] where the dumbbell figure of revolution prevails instead of the spheroidal. If this is really the case, and the evidence suggests it, then there would be a reason for the existence of the two primary centers of activity in the sun, instead of its having a single center. Some double acting system appears to impress itself generally upon the solar cosmical relations. From this we should expect to find that the sun has two magnetic and two meteorological systems, interacting so as to form the configuration of the external field as measured at the earth. There would then be sufficient ground for a differential action in the terrestrial pressures and temperatures, as detected in the discussion of such data by many students.

This view is quite in harmony with the well known fact of the existence of numerous binary systems of suns more or less widely separated, and it can not be regarded as unlikely that the sun is actually developing in this way. The enormous mass of the sun would seem to entice its constituents to group themselves preferably about two centers for the physical processes involved in circulation and radiation, rather than about one, and I suspect that this is the correct explanation of several well known phenomena.

DISCUSSION OF THE OBSERVATIONS.

On Table 1 are given some examples of the slope of the line of maximum frequency numbers in successive years. These were drawn originally by a careful examination of the entire set of figures, and an effort was made to locate the line along the maximum numbers so as to balance as nearly as possible the entire system on either side of it. Some regard was paid to the average trend of the lines in the other portions of the same zone, whereby one's judgment was guided in cases of doubt. Entire impartiality was exercised as far as practicable, and the results now about to be described were entirely unexpected. It would perhaps be preferable to utilize least square methods, if one could afford so great labor. The lines are all numbered, as 16, 17 in the zone + 50° to + 70°, which are complete; those in zone + 10° to + 30°, namely 14, 15, 16, are fragmentary on Table 1. We now count the number of days which have elapsed for a certain number of periods, in order to find the average rate of retardation per rotation of 26.68 days. Thus, for the line 16, zone + 50° to + 70°, about 12 periods elapsed, beginning with period 2 and ending with period 14, while the line was trailing, or the period was retarded, 26.7 days. Hence, 26.7 ÷ 12 = 2.225 days retardation per period of 26.68 days, so that the rotation period in that zone is 28.905 days. Similarly, line 17 gives a retardation of 26.2 days in 11 periods. Hence, 26.2 ÷ 11 = 2.382. These two values are entered in the proper place on Table 2. The results have been grouped by years where the solar energy

TABLE 2.—*Retardation of the sun in different latitudes as derived from the prominence frequency in longitude.*

Years.	Slope.	Line.	Periods.	Days.	Retardation.	Line.	Periods.	Days.	Retardation.
		Zone + 10° to — 10°.							
1871–1877	Max.–Min.	1	90	9.0	0.100	....	....	......	....
		2	90	9.0	0.100	....	....	......	....
1884–1888	Max.–Min.	3	69	6.5	0.094	....	....	......	....
		4	69	7.4	0.107	....	....	......	....
1889–1893	Min.–Max.	5	68	5.2	0.077	....	....	......	....
		6	68	6.0	0.088	....	....	......	....
1894–1900	Max.–Min.	7	96	11.4	0.119	....	....	......	....
		8	69	8.2	0.119	....	....	......	....
		Mean.........			0.101				
		Zone + 10° to + 30°.				Zone — 10° to — 30°.			
1871–1877	Max.–Min.	1	18	12.8	0.711	1	15	12.5	0.833
		2	41	28.2	0.688	2	28	22.1	0.789
		3	39	26.1	0.669	3	38	26.5	0.697
		4	35	25.3	0.723	4	39	27.0	0.692
		5	25	20.2	0.808	5	37	27.0	0.729
		6	26	17.8	0.684	6	26	19.0	0.731
		7	9	6.0	0.666	7	10	7.3	0.730
1884–1888	Max.–Min.	8	19	14.0	0.737	8	16	14.0	0.875
		9	34	26.0	0.765	9	31	27.0	0.873
		10	35	26.0	0.743	10	33	26.7	0.809
		11	35	25.0	0.714	11	35	26.5	0.803
		12	10	7.2	0.720	12	17	14.0	0.824
1889–1893	Min.–Max.	13	18	16.2	0.900	13	16	13.6	0.850
		14	31	26.7	0.863	14	34	26.8	0.788
		15	31	26.2	0.845	15	34	26.7	0.785
		16	23	19.0	0.826	16	27	22.0	0.815
1894–1900	Max.–Min.	17	33	26.3	0.797	17	33	26.4	0.800
		18	37	26.7	0.722	18	35	27.0	0.722
		19	35	27.8	0.794	19	34	26.6	0.783
		20	34	26.6	0.783	20	36	28.0	0.778
		21	28	20.7	0.739	21	35	26.8	0.766
		....	....	......	....	22	34	24.0	0.706
		....	....	......	....	23	13	9.8	0.753
		Mean.........			0.757	Mean.........			0.782

[12] Compare Figures of Equilibrium of Rotating Masses of Fluids. By G. H. Darwin, Proc. Roy. Soc. Vol. XLII. 1887, p. 359. Thomson and Tait, Nat. Phil. Vol. I, part 2, pp. 330–335.

LIBRARY OF THE UNIVERSITY OF CALIFORNIA

TABLE 2.—*Retardation of the sun in different latitudes as derived from the prominence frequency in longitude*—Continued.

Years.	Slope.	Line.	Periods.	Days.	Retardation.	Line.	Periods.	Days.	Retardation.
		Zone + 30° to + 50°.				Zone — 30° to — 50°.			
1871–1877	Max.–Min.	1	15	21.0	1.400	1	18	19.8	1.100
		2	20	27.0	1.350	2	26	27.0	1.038
		3	20	27.4	1.370	3	27	26.4	0.978
		4	19	28.0	1.474	4	25	27.3	1.092
		5	18	27.4	1.522	5	24	26.7	1.112
		6	18	27.3	1.517	6	27	27.7	1.026
		7	24	33.0	1.375	7	24	24.6	1.025
		8	21	25.3	1.205	8	10	9.9	0.990
1884–1888	Max.–Min.	9	20	24.2	1.210	9	15	14.5	0.967
		10	21	27.0	1.286	10	28	26.0	0.929
		11	22	27.2	1.236	11	27	23.6	0.874
		12	23	27.5	1.196	12	32	27.2	0.850
		13	21	27.5	1.309	13	29	27.2	0.938
		14	19	26.0	1.368	....	....	......	......
1889–1893	Min.–Max.	15	22	27.0	1.227	14	28	27.0	0.964
		16	23	27.0	1.174	15	29	27.8	0.958
		17	24	27.0	1.125	16	32	27.2	0.850
		18	24	27.5	1.146	17	27	27.2	1.007
		19	25	27.7	1.108	18	26	26.0	1.000
		20	26	27.0	1.038	....	....	......	......
1894–1900	Max.–Min.	21	27	27.4	1.015	19	30	27.8	0.927
		22	26	27.0	1.038	20	29	27.2	0.938
		23	30	27.0	0.900	21	23	26.0	1.130
		24	35	26.5	0.786	22	25	27.0	1.080
		25	28	26.2	0.936	23	29	28.0	0.965
		26	27	23.8	0.881	24	30	27.0	0.900
		Mean.........			1.192	Mean.........			0.989
		Zone + 50° to + 70°.				Zone — 50° to — 70°.			
1871–1877	Max.–Min.	1	13	27.7	2.131	1	11	20.6	1.873
		2	13	27.0	2.077	2	15	26.6	1.773
		3	14	27.0	1.928	3	13	27.0	2.077
		4	11	27.3	2.482	4	12	27.0	2.250
		5	13	27.5	2.115	5	15	27.4	1.827
		6	15	27.0	1.800	6	15	26.8	1.787
		7	19	27.3	1.437	7	9	19.0	2.111
		8	8	14.0	1.750	....	....	......	......
1884–1888	Max.–Min.	9	18	27.7	1.539	8	10	26.0	2.600
		10	18	28.0	1.556	9	10	26.2	2.620
		11	21	27.6	1.314	10	12	27.8	2.317
		12	19	27.3	1.437	11	13	26.4	2.031
		13	14	27.7	1.979	12	11	26.5	2.409
1889–1893	Min.–Max.	14	14	27.7	1.979	13	11	26.0	2.364
		15	15	27.4	1.827	14	12	26.6	2.217
		16	12	26.7	2.225	15	11	27.8	2.527
		17	11	26.2	2.382	16	11	27.0	2.455
		18	13	28.0	2.154	17	11	26.4	2.400
		....	....	......	......	18	12	27.5	2.292
1894–1900	Max.–Min.	19	15	27.0	1.800	19	13	26.0	2.000
		20	13	27.0	2.769	20	11	27.0	2.455
		21	9	26.0	2.889	21	10	27.5	2.750
		22	10	27.4	2.740	22	11	27.0	2.455
		23	10	27.5	2.750	23	15	26.4	1.760
		24	11	27.5	2.500	24	18	27.6	1.533
		25	12	27.0	2.250	25	17	27.7	1.629
		Mean.........			2.072	Mean.........			2.180

is passing from maximum to minimum, 1871–1877, 1884–1888, 1894–1900, and again where it is passing from minimum to maximum (1878–1883, lacking), 1889–1893, so as to study the effect of this variation in the retardation; but the unfortunate gap 1878–1883 prevents a satisfactory comparison between these two groups. The several zones are given separately for each hemisphere, and the successive trails can be readily scrutinized.

The first column of Table 2 contains the years of the groups; the second the slope of the 11-year curve, roughly; the third the number of the line in the zone; the fourth the number of periods elapsed; the fifth the number of days of retardation in these periods; the sixth the average retardation in days on the 26.68-day period. The mean retardation for each zone in both hemispheres is given, and has been collected in Table 3. It was necessary to assume that the mean latitude of the occurrence of the prominences is in the middle of each zone, though this can not be strictly correct. It would require very extensive computation to determine the mean latitude of occurrence of the several zones more accurately. The aspect of the path of maximum frequency as given on fig. 4 of my previous article entitled Synchronous Changes,[13] is favorable to this simple assumption.

TABLE 3.—*Mean retardation by zones.*

Mean latitude.	Retardation.			Mean period.
	North.	South.	Mean.	
°				
0	0.000	0.000	0.000	26.68
5	0.101	0.101	0.101	26.78
20	0.757	0.782	0.770	27.45
40	1.192	0.989	1.091	27.77
60	2.072	2.180	2.126	28.81

TABLE 4.—*Bigelow's rotation periods.*

Latitude.	Daily angular velocity.	Sidereal period.	Synodic period.
°	′	*Days.*	*Days.*
Pole 90	788	27.40	29.63
85	790	27.32	29.54
80	793	27.23	29.43
75	795	27.15	29.33
70	799	27.03	29.18
65	804	26.86	29.00
60	809	26.70	28.81
II Pr. 55	815	26.50	28.58
50	824	26.20	28.23
45	832	25.94	27.93
40	837	25.81	27.77
35	840	25.71	27.66
30	842	25.66	27.60
25	845	25.57	27.50
I Pr. 20	846	25.53	27.45
15	852	25.36	27.26
Spots 10	859	25.15	27.00
5	866	24.95	26.78
Equator 0	869	24.86	26.68

A careful examination of the individual determinations of the retardations in the several zones shows that there is a wide fluctuation which increases in magnitude from the equator toward the poles. In order to obtain a clear idea of the law of the retardations these results have been plotted on fig. 2.

The mean retardation, with an approximate maximum and minimum retardation, is there indicated. From the mean line I have scaled off the corresponding synodic periods for every five degrees of latitude, as given in Table 4, and have computed the sidereal period and the daily angular velocity, X, in minutes of arc belonging to them. These transformations can readily be made by interpolations from Table 5.

[13] Monthly Weather Review, January, 1903, Vol. XXXI, p. 17.

The latitude at which the maximum of spots is commonly observed, and also the latitude of the maxima I and II of prominence frequency, are indicated in Table 4 and fig. 2 by the terms "Spots," "I Pr.," "II Pr."

TABLE 5.—*Transformations of the daily angular velocity into sidereal and synodic periods.*

T= sidereal period of the sun; E= sidereal period of the earth; S= synodic period of the sun. Then we have $\frac{1}{T}-\frac{1}{E}=\frac{1}{S}=\kappa-n$.

Daily X	T	$\frac{1}{T}=\kappa$	$\frac{1}{E}=n$	$\frac{1}{S}$	S
′					
900	24.00	0.04167	0.00274	0.03893	25.69
895	24.13	0.04144		0.03870	25.84
890	24.27	0.04120		0.03846	26.00
885	24.41	0.04097		0.03823	26.16
880	24.55	0.04074		0.03800	26.32
875	24.69	0.04051		0.03777	26.48
870	24.83	0.04028		0.03754	26.64
865	24.97	0.04005		0.03731	26.80
860	25.12	0.03982		0.03708	26.97
855	25.26	0.03958		0.03684	27.14
850	25.41	0.03935		0.03661	27.32
845	25.56	0.03912		0.03638	27.49
840	25.71	0.03889		0.03615	27.66
835	25.87	0.03867		0.03592	27.84
830	26.02	0.03843		0.03569	28.01
825	26.18	0.03819		0.03545	28.21
820	26.34	0.03796		0.03522	28.39
815	26.50	0.03773		0.03499	28.58
810	26.67	0.03750		0.03476	28.77
805	26.83	0.03727		0.03453	28.96
800	27.00	0.03704		0.03430	29.15
795	27.17	0.03681		0.03407	29.35
790	27.34	0.03657		0.03383	29.56
785	27.52	0.03634		0.03360	29.76

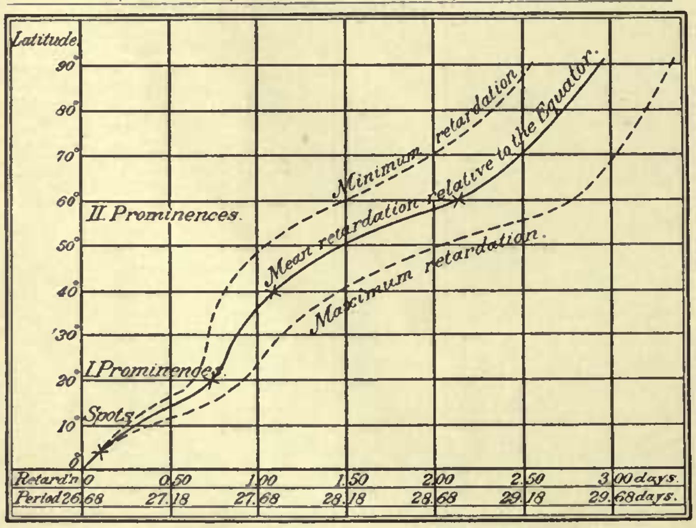


FIG. 2.—Periods of rotation of the solar photosphere derived from the prominence frequency in different zones.

It should be noted that the mean retardation does not follow a regular slope, or a simple curve that can be reduced to an analytic function. From latitude 20° to 40° there is a smaller inclination than on the slopes between 0° and 20°, or on those between 40° and 60°. In fig. 2 the line has been extended to 90°, that is to the pole, but it is unknown beyond 70°, since the polar zones were too irregular to permit any use of this method. It is probable that a continuous line, as indicated, is nearly correct.

In order to compare my result with some well known rotation periods, (taken conveniently from Miss Clerke's Problems in Astrophysics, p. 146), the following compilation is introduced:

Heliographic latitude.	Spots.	Prominences. (Bigelow).	Faculæ.
°			
0	25.09	24.86	24.66
15	25.44	25.36	25.26
30	25.81	25.66	25.48

From this it appears that my prominence rotations lie midway between those of the spots and the faculæ. Duner's rotations for the reversing layer, as quoted by Miss Clerke, are apparently impossible. The determinations of the rotation period as given by the well-known formulæ of Carrington, Spoerer, Faye, and Tisserand are found in Table 6. These periods begin to depart from the rotations as found from the prominences after leaving the latitude of 20°.

TABLE 6.—*Several denominations of the rotation periods of the solar spots in different latitudes.*

	Carrington.			Spoerer.		
d	X	T	S	X	T	S
°	′			′		
0	865	24.97	26.80	877	24.65	26.42
5	863	25.03	26.90	864	25.00	26.83
10	857	25.20	27.07	853	25.32	27.21
15	849	25.44	27.35	842	25.65	27.59
20	840	25.71	27.66	833	25.93	27.91
25	828	26.08	28.09	825	26.18	28.21
30	816	26.47	28.54	819	26.37	28.43
35	803	26.93	29.04	814	26.53	28.62
40	789	27.38	29.60	810	26.67	28.77

	Faye.			Tisserand.		
d	X	T	S	X	T	S
°	′			′		
0	862	25.06	26.90	858	25.18	27.04
5	861	25.09	26.93	857	25.20	27.07
10	856	25.23	27.11	853	25.32	27.21
15	850	25.41	27.32	847	25.50	27.42
20	840	25.71	27.66	840	25.71	27.66
25	829	26.05	28.05	830	26.02	28.01
30	815	26.50	28.58	819	26.37	28.43
35	801	26.97	29.11	806	26.80	28.92
40	785	27.52	29.76	793	27.24	29.43

It is proper to remark that the agreement in low latitudes, between the periods obtained from the prominences, the spots, and the faculæ is not unfavorable to a feeling of confidence in the results obtained by the prominence method in higher latitudes. This is perhaps strengthened by the further developments which are indicated in the next section.

THE DIFFERENTIAL CIRCULATION WITHIN THE SUN.

In order to study more minutely the meaning of the fluctuations in the relative retardations given for successive lines in Table 2, it is seen that we have practically obtained a value of the retardation for each year of the interval 1871–1900, except for the gap 1878–1883, and that by plotting these as ordinates on a diagram whose abscissas are the years, a curve of relative retardation in the several zones can be constructed. Fig. 3 exhibits these data in a graphical form. Thus, in the northern hemisphere, for the zone + 50° to + 70°, the ordinates in Table 2, beginning with that for 1871, read 2.13, 2.08, 1.93, ... 2.25, and these form the successive points of the retardation curve.

In the upper section of the diagram marked "Prominence frequency" is reproduced the curve of average prominence frequency for the entire sun, which is the mean curve of the zonal system shown on fig. 2 of my paper on Synchronous Changes,[14] and is also reproduced at the head of fig. 28 of my paper, A Contribution to Cosmical Meteorology.[15] An inspection of the curves of fig. 3, shows plainly three important facts of fundamental significance: (1) the retardations relative to the equatorial period of rotation, 26.68 days, increase toward the poles; (2) the irregularities in the observed retardations are very much greater in the polar than in the equatorial zones; (3) these irregularities in the retardation do not appear to be accidental, but they synchronize closely with the variations in the frequency of the prominences. The value of this last inference is very great, in view of the other facts brought out in various portions of my research. Using this prominence curve as the standard of reference we have already proved the following facts: (1) The elements of the earth's magnetic field fluctuate with it annually in synchronism; (2) the terrestrial temperatures and barometric pressures synchronize with it, as will be shown conclusively in my next paper, in the Monthly Weather Review for November, 1903; (3) the internal circulations of the sun, as recorded in the rotational velocities of the photosphere, also synchronize with the same curve. This exhibit binds the entire solar and terrestrial atmospheres in one synchronous circulation, and it therefore places the entire subject of cosmical meteorology upon a satisfactory basis, entirely in harmony with the procedure marked out in previous papers.

While it can not be supposed that this discussion of the solar prominence frequency in longitude gives us final quantitative results on the rotation phenomena of various zones, yet the line of argument is sufficiently sustained to warrant further extensions of the research. We have shown that the solar angular velocity diminishes from the equator toward the poles at a certain rate, as on fig. 1 for example, or as on fig 4.

This is in harmony with the von Helmholtz-Emden equations for a rotating mass hot at the center and cooling toward the surface.[16] In such a mass there are discontinuous concave cylindrical surfaces coaxial with the axis of rotation, the equatorial parts being nearer the axis than are the polar parts. This also implies that the polar regions of the sun are warmer than the equatorial by reason of the currents from the center toward the poles. At a surface of discontinuity, on each side of which the pressure is the same, but the temperature and angular momentum different, as where a rapidly moving current flows over a more slowly moving current in the earth's atmosphere, the conditions are favorable for forming vortex tubes, terminating on the surface, but extending through the mass of the sun. They are right-handed in the northern hemisphere and left-handed in the southern hemisphere, for convective actions from the equator toward the poles. If vortices are thus formed in the sun, so far as the state of its material permits, then the solar mass is in fact in a polarized state, the internal matter tending to rotate throughout the globe around such lines as are the generators of the required discontinuous surfaces. The turbulent conditions of internal circulation tend to a lawful disposition by the regulative action of a hot mass gravitating to a center by its own internal forces and emitting heat through these processes of circulation accompanied by polarization and rotating vortex tubes. The contents of a tube must be made up of molecules and atoms more or less charged with electricity, and the necessary rotatory motion produces Amperean electric currents which are a sufficient cause for the generation of a true magnetic field, positive on the northern and negative on the southern hemisphere of the sun. This conforms to the result reached years ago by my analysis of the terrestrial magnetic field, which showed that the earth appears to be immersed in a magnetic field perpendicular to the plane of the ecliptic and positive to the north of it. Variable circulation within the solar mass would display itself in corresponding changes in the rotation of the discontinuous surfaces, in the vortices carrying electrical charges, in the external magnetic field, in the number of prominences, faculae, and spots, in the earth's magnetic and electric fields, and in the terrestrial temperatures and pressures. Synchronism having thus been established throughout this vast complex cosmical system and referred back to fundamental thermodynamic and hydrodynamic laws, it becomes possible to make further advances in the problems of solar physics. Thus, the curvature of the internal lines can

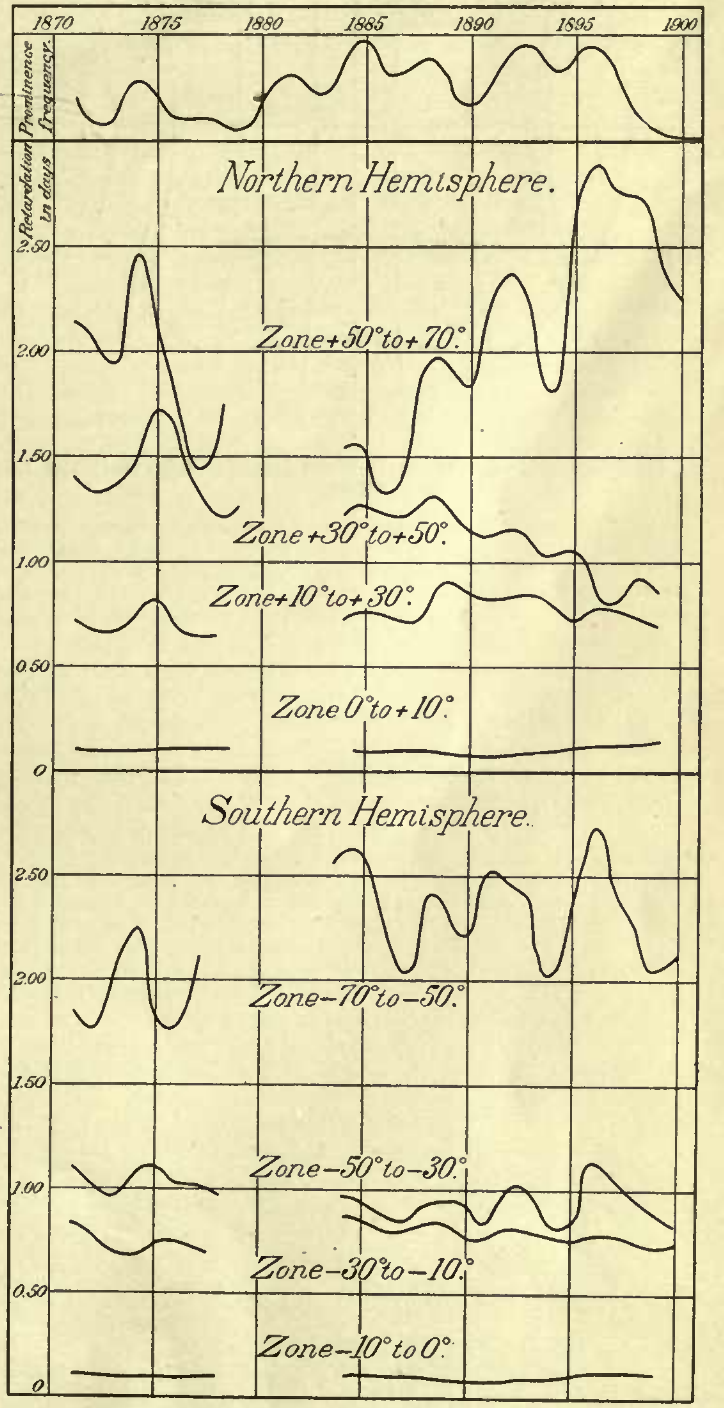


Fig. 3.—Variable retardations in the periods of rotation of the solar photosphere.

[14] Monthly Weather Review, January, 1903, Vol. XXXI, p. 10.
[15] Monthly Weather Review, July, 1902, Vol. XXX, p. 352.
[16] See Eclipse Meteorology, pages 70 and 71.

be studied in different parts of the meridian section on passing from the surface of the sun to internal parts by means of the vortex law of constant angular momenta, $\Omega = \omega \varpi^2$, under the assigned thermal conditions. We shall make an attempt to do this in a report which will contain the tabular data in full upon which these deductions are based.

If it is true that large cosmical cooling masses in rotation

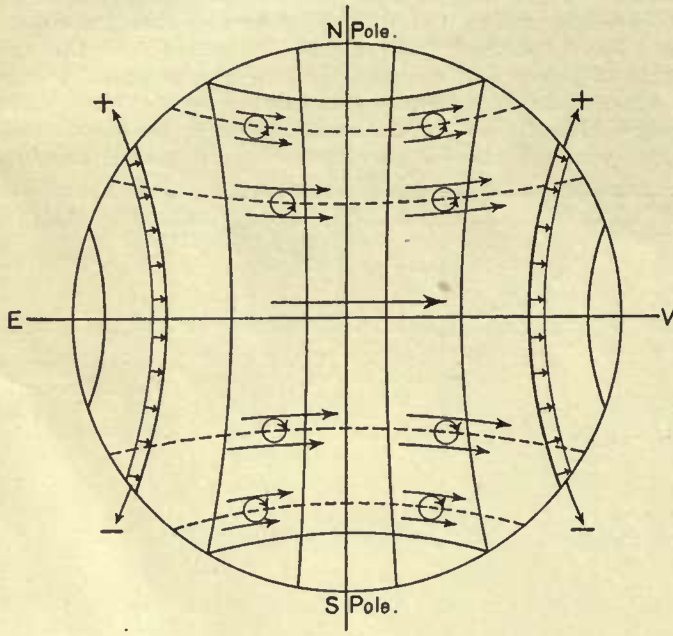


Fig. 4.—Formation of vortices in the solar mass by differential rotations.

contain a polarized or vortical internal structure which is the basis of a magnetic field, then it follows that this is the explanation of the earth's magnetism as well as of the magnetism of the sun. Hence, all stars are magnetized spheres, and their relative magnetism would be a measure of the activity of their internal circulations. Thus, the relative intensity of the earth's and the sun's magnetization becomes a measure of the internal vortical circulation in polarized tubes, and the variations of the earth's magnetic field have a cosmical significance, not only as to the direct action of the sun as a great rotating variable magnet, but as a measure of the forces which go to make up the solar output in several manifestations of energy. The summary of this line of thought may be found in chapter 4 of my "Eclipse Meteorology." It is proper to renew my objection to the results derived by other investigators for any solar rotation period which is shorter than 26.68 days, because it does not seem to be possible in view of the above analysis of solar conditions. Thus, we must reject Spoerer, 26.32; Broun, 25.92, 25.86, and 25.83; Hornstein, 26.39, 26.03, 26.24, and 25.82; Liznar, 26.05 and 25.96; Müller, 25.66, 25.79, 25.86, 25.87, and 25.47; von Bezold, 25.84; Hamberg, 25.84; Ekholm and Arrhenius, 25.93; Schuster, 25.809 or 25.825. The numerous computations, giving results so widely different from that apparently ruling in the sun as derived from observations upon its own material, seem to indicate that the application of these several methods of computation to terrestrial data raises grave doubts as to their value. There are numerous difficulties in applying least square methods to solar-terrestrial data in the present state of our science. The great fluctuations going on within the solar mass tend to mask the fundamental law until it has been derived, at least approximately, by simpler methods. But the evidence is very positive that the equatorial period of 26.68 days is the shortest one actually prevailing in any portion of the mass of the sun.

II.—SYNCHRONISM OF THE VARIATIONS OF THE SOLAR PROMINENCES WITH THE TERRESTRIAL BAROMETRIC PRESSURES AND THE TEMPERATURES.

SEVERAL OPINIONS ON THE SUBJECT OF SYNCHRONISM.

The numerous studies during the past fifty years into the apparent synchronism between the solar variations of energy and the terrestrial effects, as shown in the magnetic field and the meteorological elements, have been on the whole unsatisfactory, if not disappointing. Just enough simultaneous variation has been detected in the atmospheres of the sun and the earth to fascinate the attentive student, if not to justify a large expenditure of labor, in view of the great practical advantages to be obtained in the future as the result of a complete understanding of this cosmical pulsation. The attack upon the problem has really consisted in rather blindly groping for the most sensitive pulse in the entire cosmical circulation, and in disentangling the several interacting types of impulses. It is evident that the partial failures hitherto attending this work have been due to two principal causes: (1) The comparison was made between the changes in the spotted areas of the sun and the terrestrial variations, but these solar changes were not sensitive enough to register a complete account of the action of the solar output. Discussions of the spots are being replaced by others upon the solar prominences and faculæ, which respond much more exactly to the working of the sun's internal circulation. (2) The magnetic and the meteorological observations have not been handled with sufficient precision to do justice to the terrestrial side of the comparison. It is evident that all these physical data at the sun and at the earth must be computed with an exactness comparable to that of astronomical observations of position, if meteorology is to be raised to the rank of a cosmical science. When one considers the crudeness of the meteorological data, taken the world over, due to the character of the instruments employed, the different local hours of observation, and the divergent methods of reduction, it is no wonder that the small solar variations have been swallowed up in the bad workmanship of meteorologists. The prevailing methods have been sufficient for forecasting and for climatological purposes, but they are entirely inadequate for the cosmical problems whose solution will form the basis of scientific long-range forecasts over large areas of the earth, that is, for forecasting the seasonal changes of the weather from year to year. It is perfectly evident that if secular variations of any kind, such as the annual changes in terrestrial pressure, temperature, or magnetic field, are to be attributed to solar action, the original observations must be finally reduced to a homogeneous system. The local peculiarities of each station must be carefully eliminated, and the data of numerous stations must be concentrated before anything like quantitative cosmical residuals can be obtained. When we consider that there have been numerous changes in the elevations of barometers, various methods of reducing the readings, and many groups of selected hours of observations entering into the series at the same station, how could it be expected that any thing better than negative results in solar problems would be obtained? The skeptical attitude of conservative students, who declare that the many indecisive results already obtained mean that there is no true and causal solar-terrestrial synchronism, is, of course, quite fallacious until it has been demonstrated by the use of first-class homogeneous data that the suspected physical connection is imaginary. There is but little question that the existing uncertainty is in fact based upon the use of the very imperfect methods of observation and reduction which have prevailed in meteorological offices, rather than upon the unreality of the phenomena in nature. At present the difficulties of the research are diminishing by reason of two improvements; (1) a better knowledge of where to make the comparison, and (2) the gradual acquisition of reliable secular data. Thus, the prominence data are superseding the sun-spot numbers, and it has now become comparatively easy to traverse the magnetic and the meteorological fields with our improved standard curve of comparison, and to bring out the fundamental typical synchronism in nearly every series of observations, so far as the annual means are concerned.

The importance of emancipating this subject from the prevailing skepticism is evidently in the interests of advancing cosmical science. If we can prove that other forces than the Newtonian gravitation and radiation are interacting between the sun and the earth, it becomes a conclusion of vital interest to astronomers. As an example of the present state of opinion, we note Prof. Simon Newcomb's address[17] before the Astronomical and Astrophysical Society of America on December '29 1902, in which he says:

> The conclusion is that spots on the sun and magnetic storms are due to the same cause. This cause can not be any change in the ordinary radiation of the sun, because the best records of the temperature show that, to whatever variations the sun's radiation may be subjected, they do not change in the period of the sun spots.

We shall, on the other hand, show in this paper that terrestrial temperatures do, as a whole, change with the variations of the solar prominences, and this will tend to modify Professor Newcomb's inference. The question whether the connection is direct or indirect, by a magnetic field or by some special action of radiation, is to be decided finally by an appeal to the observations. Dr. J. Hann writes in his Lehrbuch der Meteorologie, pages 626, 627:

> These can lead to the discovery of the period, but it is very difficult to find the true length of the period, since the amplitude of the variation of the meteorological elements within the period is not very great, because so many other influences are present, which stand in the way of deriving more accurate mean values out of long intervals of time. As yet no one has succeeded in surely deducing for any one meteorological element a cyclic variation of considerable amplitude.

These efforts have been applied to variations of temperature, clouds, rainfall, thunderstorms, hail, barometric pressures, cyclones, and winds, especially with the view of finding an 11-year period synchronous with that of the sun spots. It should be noted that a shorter period, of about three years, is probably the better period of synchronism to be studied. Also, synchronous movements need not be truly periodic. Indeed, there may be true correspondence with very irregular and aperiodic changes. It is easier to connect loosely constructed variations in the prominences of about three or four years duration with terrestrial variations than to establish synchronism in the 11-year sun-spot period. Dr. A. Sprung, in his Lehrbuch der Meteorologie, pages 366, 367, writes:

> Therefore, a connection between the sun-spot frequency and the changes in our atmosphere can not well be denied. It is probable that the pe-

[17] Science, January 23, 1903.

riodic changes in the atmosphere are not caused directly through the sun spots, but that both phenomena are brought about through one common or by several interacting causes, whereby a displacement of the periods relative to one another becomes possible.

Prof. Cleveland Abbe has frequently expressed in the Monthly Weather Review a very doubtful view regarding the advisability of such researches, with the object of discouraging further efforts to unravel the solar-terrestrial net. Thus, in the Monthly Weather Review for June, 1901, page 264, he writes:

As the periodicities in sun spots, the width of the spectrum lines, the magnetic and auroral phenomena are sufficiently well marked to be satisfactorily demonstrable, while corresponding variations in pressure, temperature, wind, and rainfall are small, elusive, and debatable, we must caution our readers against being carried away by optimistic promises. It is certainly impressive to the thoughtful mind to realize that there is even a slight connection between solar and terrestrial phenomena, but the delicacy of this connection is such that it still remains true that the study of meteorology is essentially the study of the earth's atmosphere as acted upon by a *constant source of heat* from the sun. None of these astrophysical studies should tempt the meteorologist to wander far from the study of the dynamics of the earth's atmosphere and the effects of the oceans and continents that diversify the earth's surface.

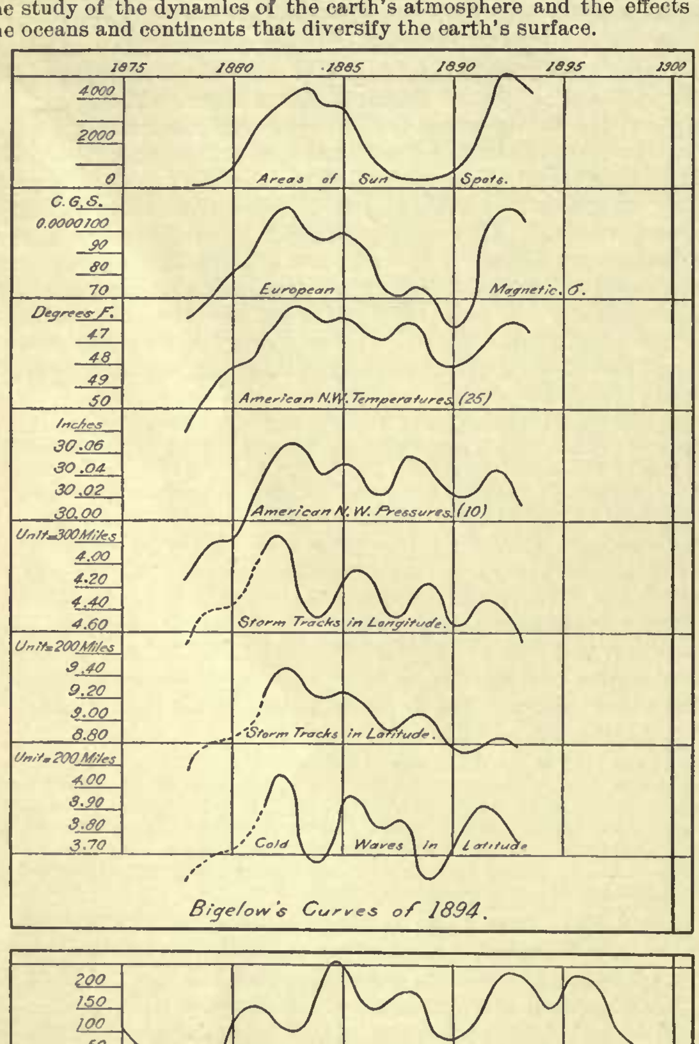


Fig. 5.—Solar and terrestrial synchronism.

We have, nevertheless, merely to recall the works of many scientists in order to realize how strong a hold this problem has upon the astrophysical meteorologist: Herschel, 1800; Gautier, 1844; Fritsch, 1854; Arago, 1855; Zimmermann, 1856; Wolf, 1859; Meldrum, 1870; Koeppen, 1873; Hill, 1880; van Bebber, 1882; Blanford, 1889; Bruckner, 1890; Lockyer, 1898; Carrington, Spoerer, Wolfer, and many others. The number of students who are taking up the problems of cosmical meteorology is rapidly increasing, and this shows that there is encouragement for such work.

The present paper continues the discussion of an investigation first published in 1894,[18] which brought out the fact that there is a synchronous variation in short cycles of about three years duration superposed upon the 11-year sun-spot period. In Bulletin No. 21, Solar and Terrestrial Magnetism, page 127, it was said:

A comparison of the mean American meteorological curve with the European magnetic curve certainly shows conformity to such an extent as to exclude merely accidental physical relations. Should such a result be obtained also in the future, it will be a demonstration of the synchronism of the two systems of forces under consideration.

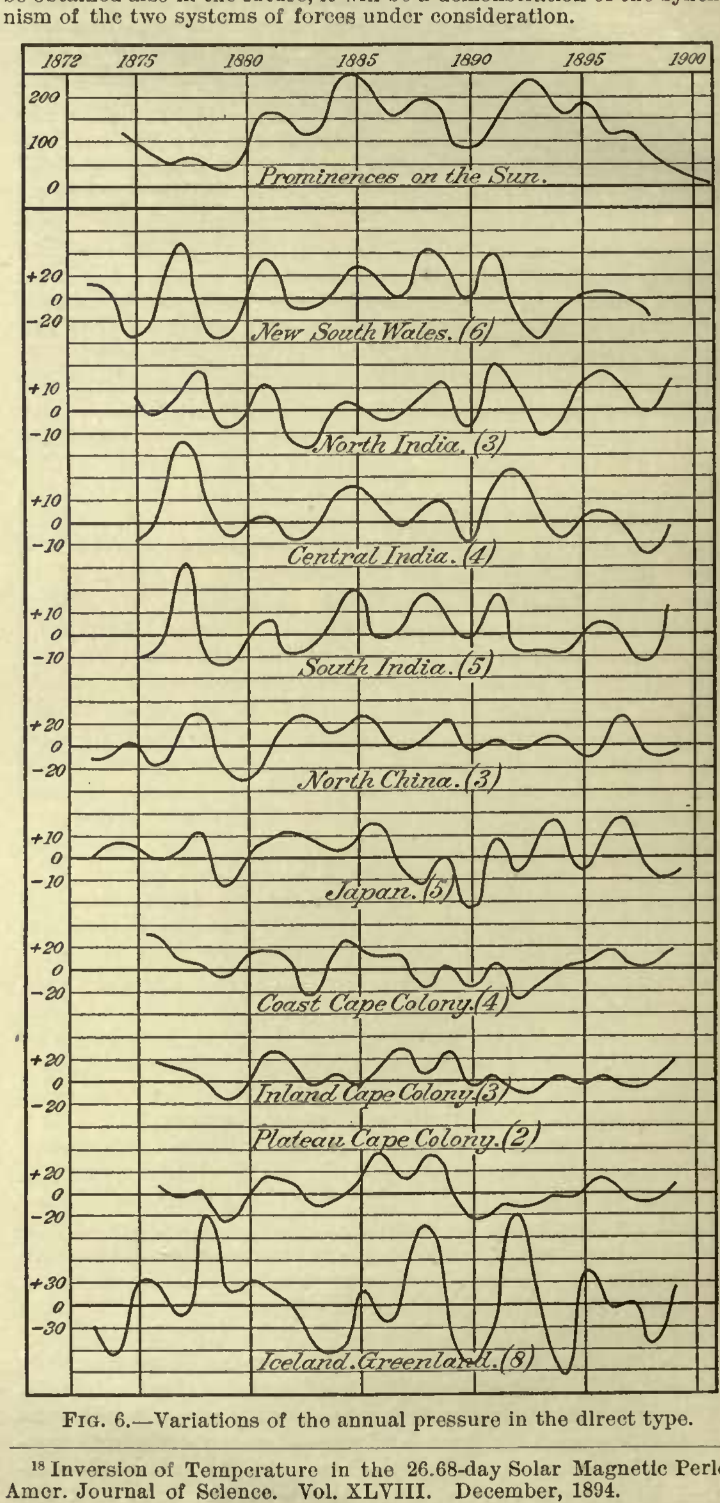


Fig. 6.—Variations of the annual pressure in the direct type.

[18] Inversion of Temperature in the 26.68-day Solar Magnetic Period. Amer. Journal of Science. Vol. XLVIII. December, 1894.

Since that time advances have been made as follows:

The magnetic curve has been extended from 1841 to 1900; the barometric pressures of the United States have been reduced to a homogeneous system; the curves of prominence frequency on the sun have been computed by Lockyer and independently by myself; the variations of the prominences have been closely associated with the changes in the angular velocity of the solar surface rotations in different zones, especially in the polar latitudes; the type of internal circulation necessary to produce this polar retardation, and to transform the solar mass into a polarized magnetic sphere, has been indicated

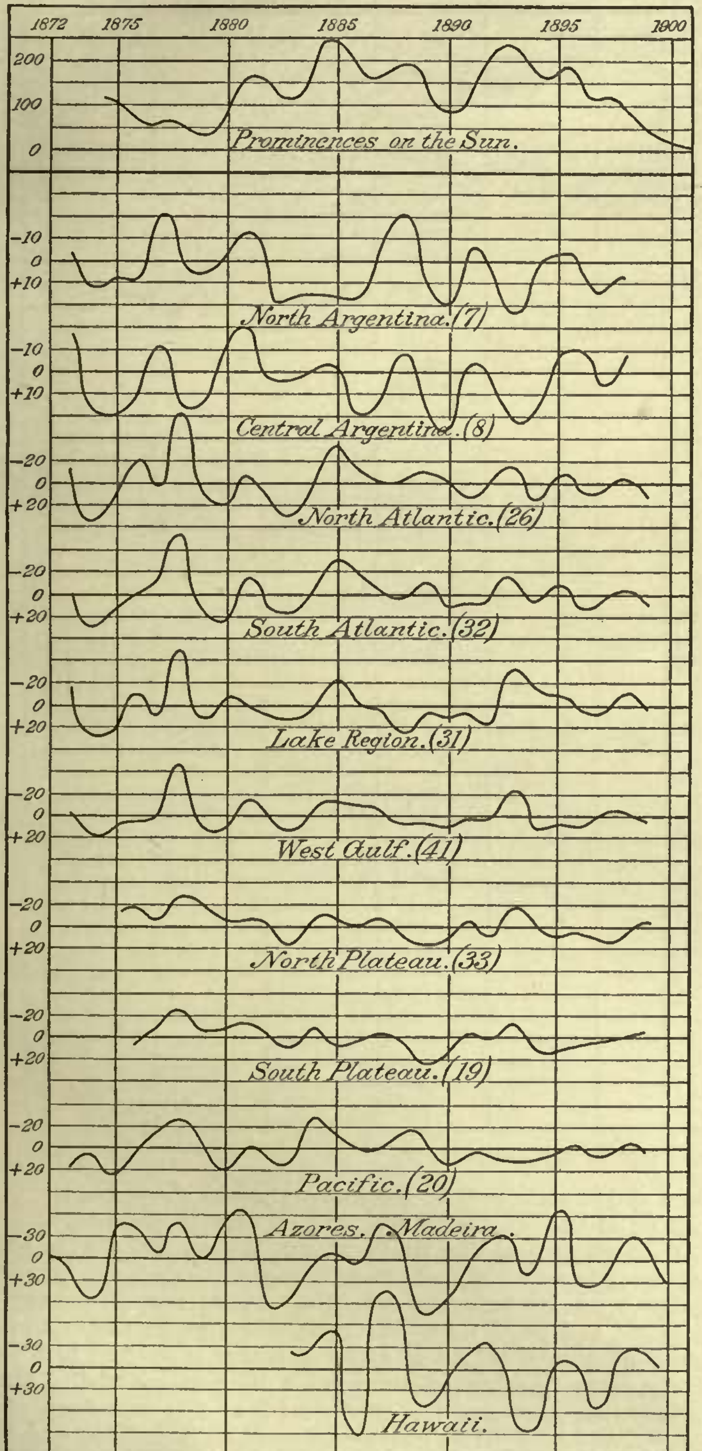


FIG. 7.—Variations of the annual pressure in the inverse type.

In the present paper we shall show the results of a discussion of the annual residuals of pressure and temperature in all parts of the earth. These have a variation in *direct* synchronism with the prominences, in certain parts of the earth, but under special conditions of orography the synchronism is of the *inverse* type. This chain of evidence is strong enough to induce confidence in regard to the fact that this solar-terrestrial physical synchronism really exists.

THE UNSATISFACTORY STATE OF THE OBSERVATIONAL DATA.

The two prevailing difficulties in extracting suitable data from the published reports of meteorological observatories, and reducing them to a homogeneous system, are the numerous changes in the elevation of the barometers, and in the very different hours of making the observations. Without the expenditure of labor entirely beyond the capacity of a single office to bestow upon the task, when the research for synchronism is extended to the entire earth, it has been necessary to

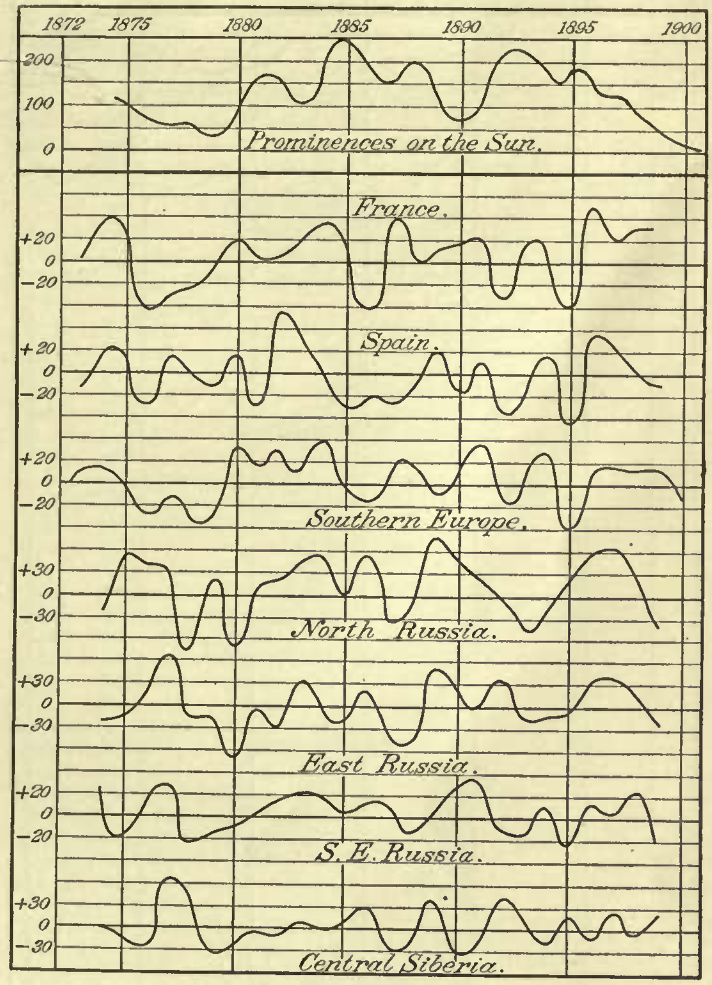


FIG. 8.—Variations of the annual pressure in the indifferent type.

use some simple devices for the sake of arriving at approximately homogeneous residuals. The work for the United States is complete for the pressures, and is in progress for the temperatures. By inspecting my Barometry Report[19] it is easy to see the reason for the necessity of the reduction. In order to give some idea of the state of the data in other countries, we note the following with respect to the barometric pressures:

For Russia-Siberia, several stations changed elevation more than once.

India, there are numerous changes of elevation.

South Africa, numerous changes of elevation, and also of the hours of observation.

New South Wales, the monthly means of observations alone

[19] Report of the Chief of the Weather Bureau, 1900–1901, Vol. II.

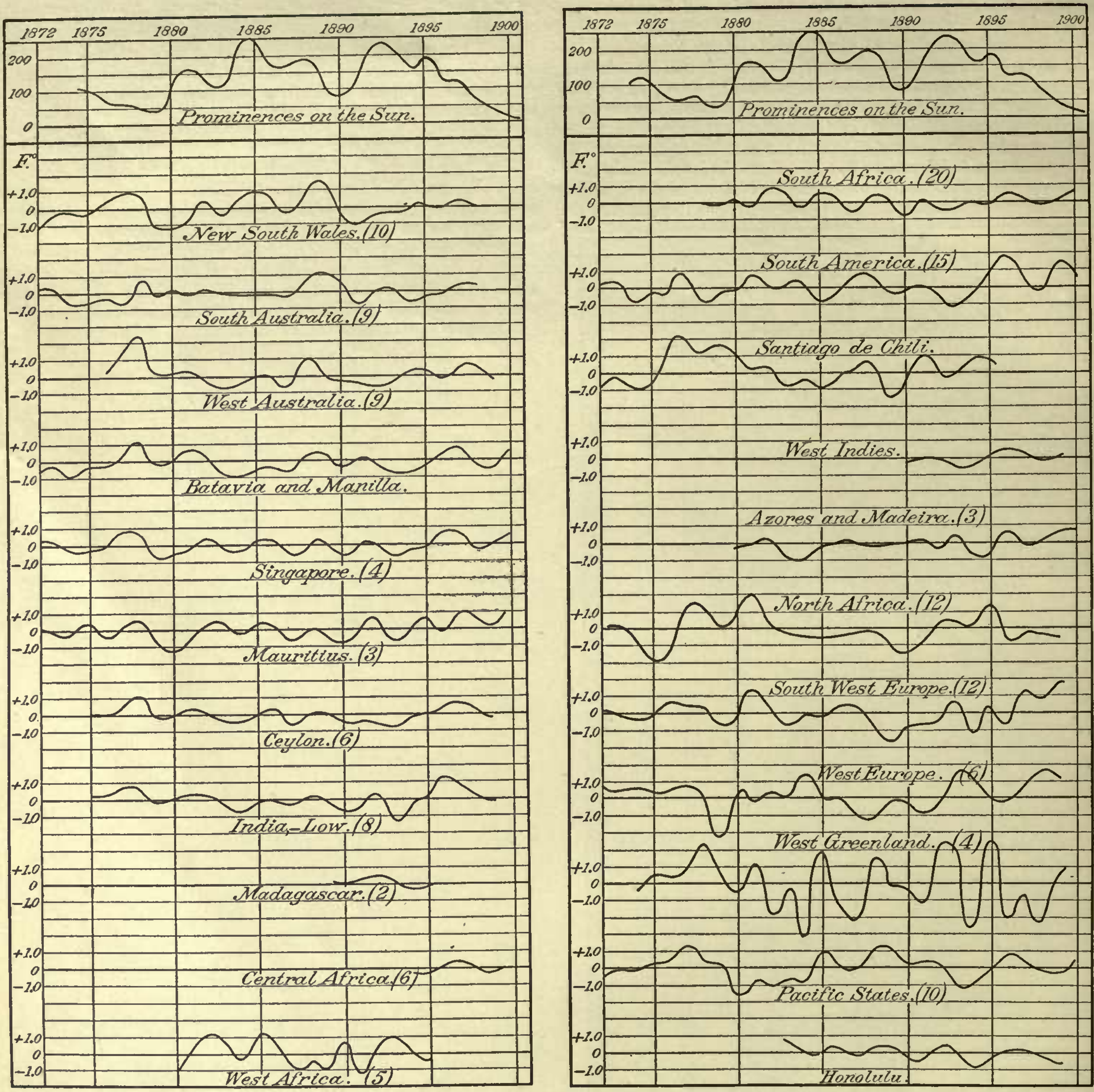


FIG. 9.—Variations of the annual temperature in the direct type.

were published. These had to be collected before the annual means could be computed.

Argentina, the monthly means of observations alone were published, and these also had to be collected before the annual means could be computed. The stations have quite short records.

Iceland and Greenland, very few changes in elevation, but not long records.

In general all the annual pressure curves were plotted, and a mean pressure and normal gradient were determined, from which the amplitude variations were taken off as residuals. Since our purpose was simply to secure the most probable annual residuals this graphic method was substituted for the exact computations which ought to be made. Frequently the secular gradient slope was so prominent throughout the series for a single station as to suggest a gradual change in the correction of the barometer relative to a normal standard.

With respect to the temperatures, the annual means were extracted from the reports, and the mean values for the several series were computed, so far as they were apparently homogeneous, and from these the residuals were formed. As the cosmical annual variation of temperature is only 1° to 2° F., it was often possible to break up a long series at the same station into homogeneous sections; but this was done cautiously, and only after clear evidence of a discontinuity in the local conditions. The great difficulty with the temperature data consists in the numerous hours of observation that have been adopted, or in the numerous selected groups of hours from which the means were derived. Many of these differ-

ences arose from artificial attempts to obtain an approximately correct 24-hour mean, to which in fact all meteorological data should be very carefully reduced. Some of the combinations of hours used are as follows:

United States, Washington mean time, 7:35, 4:35, 11:35; 7:35, 4:35, 11:00; 7, 3, 11. Seventy-fifth meridian time, 7, 3, 11; 7, 3, 10; 8, 8; maximum, minimum.

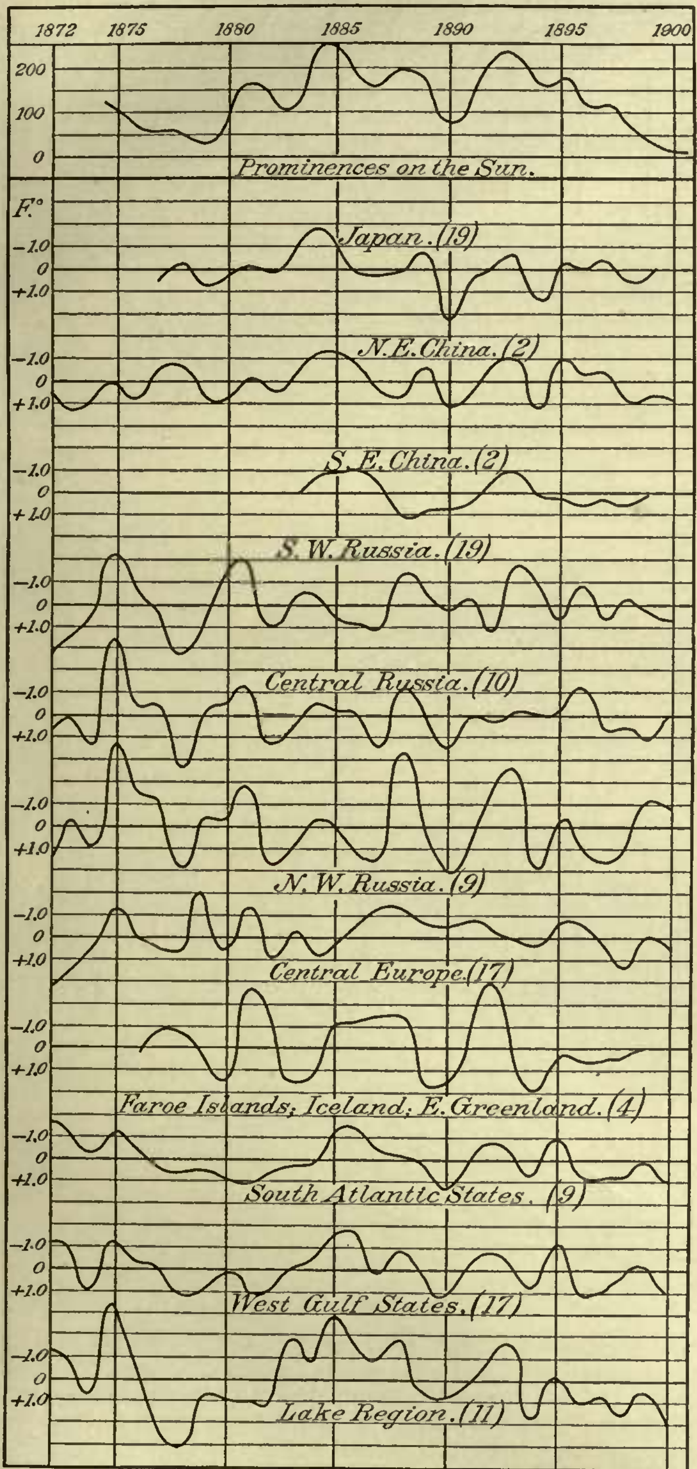


Fig. 10.—Variations of the annual temperature in the inverse type.

New South Wales, 9 a. m.; 9, 3, 9; maximum, minimum.

South Australia, 9, 3, 9; 9, 12, 3, 6, 9; maximum, minimum.

West Australia, 9, 3; 9, 12, 3; 9 a. m.; 6, 6; maximum, minimum.

Ocean Islands, hourly; 9, 3, 9, minimum; 6, 9, 1, 3, 3:58.

Japan, 9:30, 3:30, 9:30; 4-hourly, or 2, 6, 10, 2, 6, 10.

China, hourly; 10, 4, 10.

India, 8, 10, 4; 10, 4; 6-hourly, or 10, 4, 10, 4; 9:30, 3:30; 9, 4; 10:30, 3:30; maximum, minimum.

Russia-Siberia, 7, 1, 9; 7, 2, 9; 9, 12, 9; 8, 1, 9; hourly.

Europe, 7, 2, 9, 9; 7:45, 8; 6, 2, 10; 3-hourly; maximum, minimum; 7, 10, 1; 4, 7, 11; 7, 1, 7; 6, 9, 12; 3, 6, 9; 6, 12, 9; hourly.

Azores-Madeira, 9, 3, 9.

North Africa, 7, 2, 9; 7, 11, 2, 5; 7, 1, 6; 9, 3, 9.

South Africa, 6, 12, 6; 6, 2; 9, 9; 8, 8; 8 a. m.

South America, 7, 2, 9; hourly.

Iceland-Greenland, 8, 2, 9.

From such an exhibit it is no wonder that meteorology has not yet contributed its proper share to accurate cosmical physics. It is needless to recount the reason for this state of affairs, but only to urge as speedy a remedy as is possible. It might be argued that no results can be derived from such data; but this is not true, as a study of the residuals summarized in this paper amply confirms. It is, perhaps, surprising that valuable results can be extracted from the data, and this only proves how important such work might be made if sufficient care were exercised in selecting the hours of observation, and establishing rigorous methods of reduction. It frequently happens that at a given station the same hours continue to be used for many years, so that in effect its own residuals are nearly homogeneous. The means of the various combinations of selected hours generally approximate a true 24-hour mean, so that on the whole there is something like homogeneity in the different changes. The fact that residuals synchronous with solar variations actually survive, is a satisfactory evidence that the causes producing them are solar and not local terrestrial.

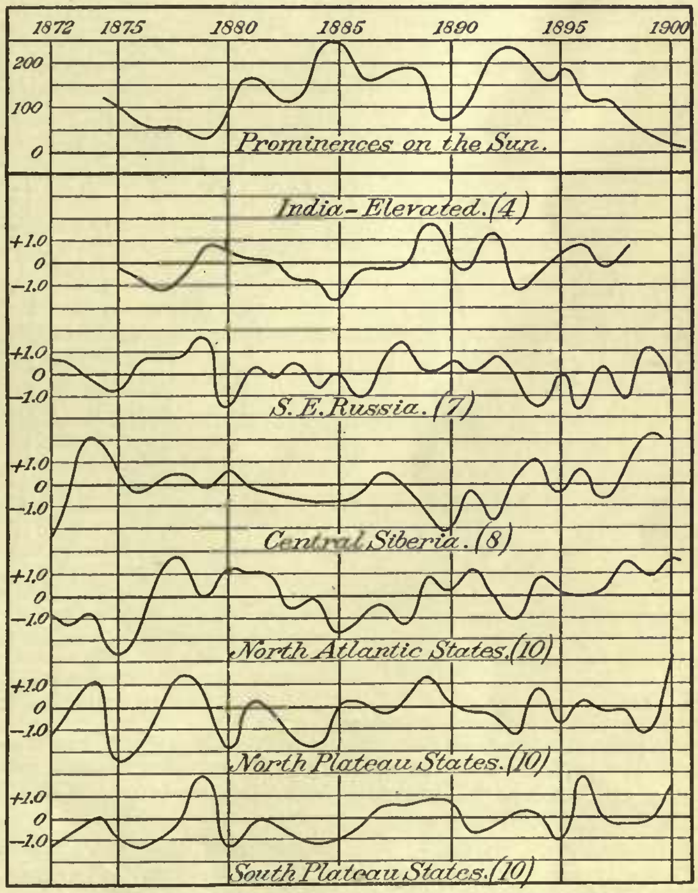


Fig. 11.—Variations of the annual temperature in the indifferent type.

It is not possible to print in the Monthly Weather Review the table of residuals for each station, and we must confine

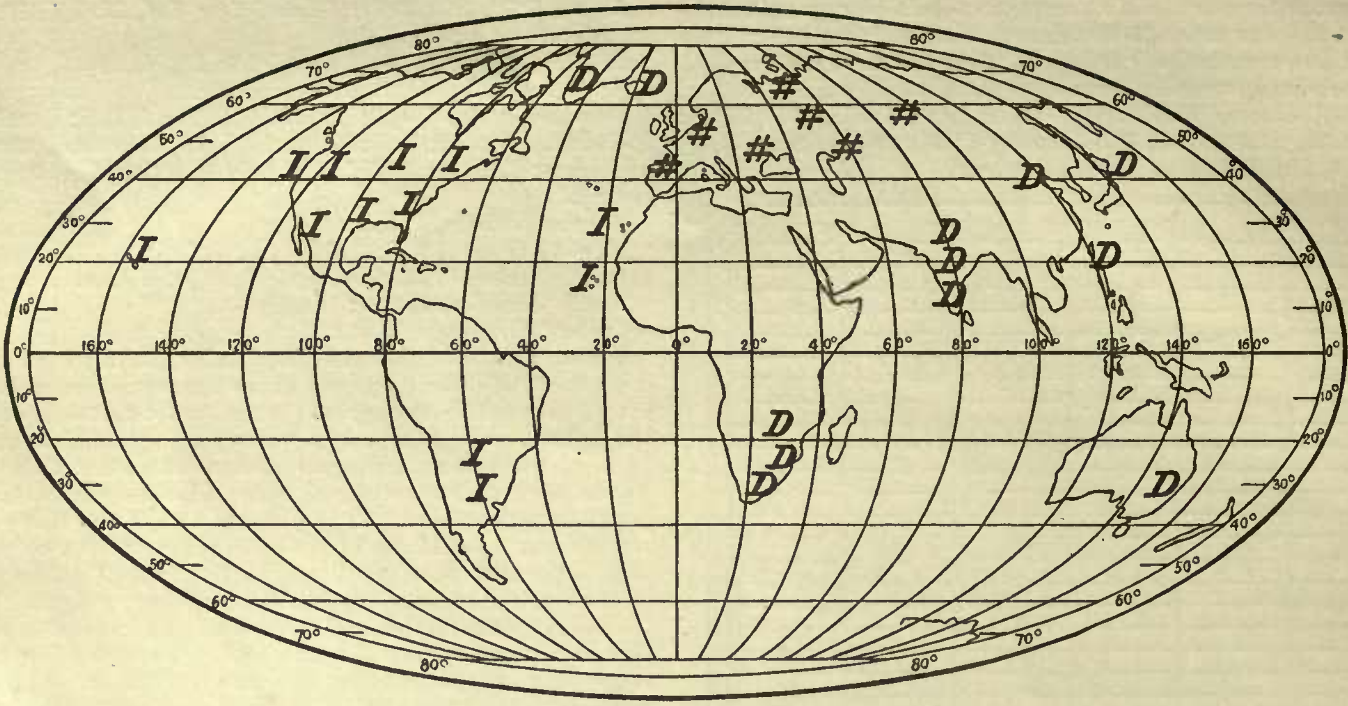


Fig. 12.—Distribution of the pressure types.

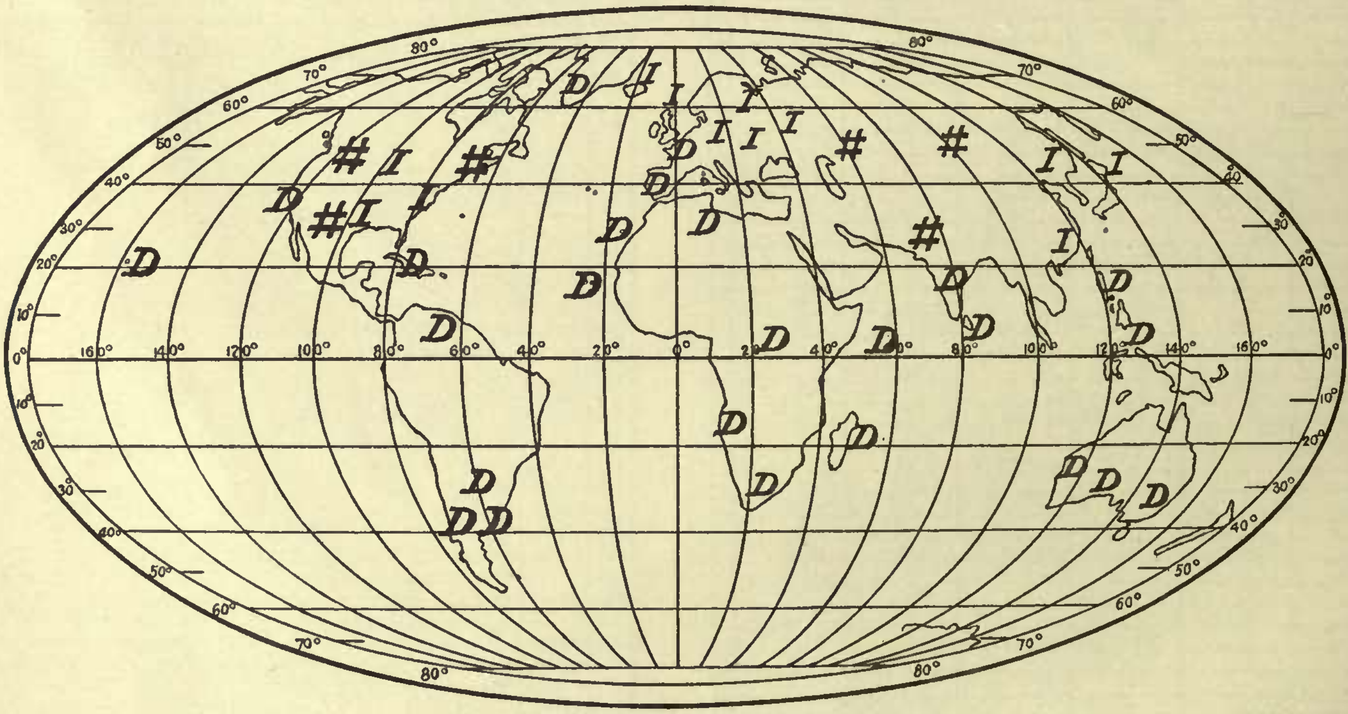


Fig. 13.—Distribution of the temperature types.

ourselves to the curves representing the mean residuals for a group of stations, the number being entered in connection with the name of the country. Thus, for New South Wales the pressure curve, fig. 6, was determined from six stations, Albany, Bathurst, Deniliquin, Goulborn, Newcastle, Sydney.

RESULTS OF THE OBSERVATIONS.

The argument for solar and terrestrial synchronism may be recapitulated as follows:

Bigelow's curves for 1894 showed a synchronism in a short period of about three years, superposed upon the 11-year sun-spot curve, for the following elements: Terrestrial magnetic field, American temperatures, pressures, storm tracks in longitude and latitude, and cold waves in latitude. In 1902 Lockyer worked out the annual variation in the solar prominences and arrived at the same system of minor crests in the sun that had previously been determined at the earth. These curves are shown on fig. 5, "Solar and terrestrial synchronism."

A study of the temperature and the pressure residuals for the entire earth shows that the phenomena of inversion prevails in the earth's atmosphere, localizing the effect of solar action in two typical curves which are the inverse of one another. I have previously found a form of inversion of energy in the terrestrial magnetic field, and efforts have been

made to explain the phenomenon. Besides the secular inversion here illustrated, I have found a semiannual inversion in the meteorological elements of the United States, as stated in other places, and much work has been done in developing this important fact.

We have treated the secular inversion as follows: The curves of the mean residuals of the pressures and temperatures, taken by geographical groups as indicated, were plotted to scale and compared with the Lockyer solar prominence curve as to the recurrence of the successive maxima and minima. They were then associated in three groups, as follows:

I. Direct type, wherein the solar and the terrestrial maxima closely match each other throughout the interval 1873–1900.

II. Inverse type, wherein the terrestrial curves must be inverted to make the maxima coincide.

III. Indifferent type, wherein there is not sufficient evidence of conformity with the type curve to be satisfactory.

There may be differences of opinion as to the assignment of some of these curves, but the reader can make any different arrangement that he prefers. It seems to me that the general fact of synchronism is so pronounced as to call for the careful consideration of meteorologists. Fig. 6, "Variations of the annual pressure in the direct type;" fig. 7, in the "inverse type;" fig. 8, "indifferent type;" fig. 9, "Variations of the annual temperature in the direct type;" fig. 10, in the "inverse type;" and fig. 11, in the "indifferent type," are sufficiently explicit without further explanation. The unit for the pressure variation is 0.001 inch, and that for the temperature is 1.0° F. The average range in annual pressure amplitude amounts to as much as 0.060 inch and that for the temperature to 2° or 3° F, more or less.

DISCUSSION OF THE LOCAL INVERSIONS.

These suggestive curves deserve more discussion than is possible in this connection, but fuller data and further remarks will be found in a forthcoming report, which will contain the original data in full. It may be desirable to call attention to the geographical distribution of the types of synchronism thus indicated, by plotting on world charts *D*, *I*, and *#*, respectively, for the direct, inverse, and indifferent types. Fig. 12, "Distribution of the pressure types," shows that, taking the earth broadly, the region around the Indian Ocean gives direct synchronism, South America and North America give inverse synchronism, while Europe and Siberia give an indifferent type. Greenland and Iceland seem to have direct type like the Indian Ocean. Fig. 13, "Distribution of the temperature types," shows that there is synchronism of the direct type for the Indian Ocean, Africa, South America, the West Indies, and the Pacific islands generally—that is to say, throughout the Tropical Zone. The inverse or the indifferent types prevail in Asia, Europe, and North America generally—that is, throughout the North Temperate Zone.

Taking the earth as a whole, the temperatures synchronize directly with the solar energy in the Tropical Zone, and inversely in the temperate zones. The indifferent type prevails in the plateau districts of the continental areas, probably because the solar type is there so broken up by the local climatic conditions as to practically obscure the synchronism. In the pressures the Eastern Hemisphere tends to direct synchronism, except in Europe and Russia, where the indifferent type prevails, and the Western Hemisphere to the inverse type. It may not be practicable to explain all that this means, but apparently we are dealing with the complication caused by superposing an atmosphere in circulation upon the unequally heated surface of the earth. The surging of the atmosphere as a whole from one hemisphere to the other, or from the continents to the oceans, is concerned in producing these effects. The trend of the great mountain systems strongly differentiates the circulation of the lower strata. Thus, the Himalaya Mountains, running east and west, check the flow of air from the Tropics to the Asiatic Continent, while the Rocky Mountains and the Andes system favor the flow along the meridians, especially in the United States. As a result, the number of cyclones crossing the United States is many times the number crossing Siberia, which is in fact singularly deficient in cyclones. South America shows a similar defect in circulation, because it lies too near the Tropical Zone.

The United States is covered by an active circulation between the Tropics and the north Polar regions, Siberia by a stagnant atmosphere, and Europe generally by a mixed and indifferent circulation, since the American cyclones tend to break up upon the territory of Europe after crossing the Atlantic Ocean. Hence, the region about the Indian Ocean is favorable for detecting direct synchronisms of pressure and temperature with the solar prominences by reason of its quiescent atmosphere, and the United States is well placed to respond to an inverse synchronism, by reason of its active circulation with a pronounced component from the north Polar regions. Europe does not possess an atmosphere which registers the solar and terrestial synchronism in a very efficient manner. This may account for the fact that the European attempts to establish a definite synchronism have issued generally with negative results. As has already been suggested, too much emphasis has been put upon the failures to make out the connection between the solar and the terrestrial synchronisms.

It should be noted that C. Nordmann[20] and A. Angot[21] deduced for certain tropical stations small residuals of temperature which are *inverse* to the sun-spot curve, but apparently synchronous. These authors have smoothed their curves by grouping successive years, and have reached small residuals. Since synchronism should display the annual variations intact, as given above, it may be questioned whether any process for eliminating the minor deflections from year to year is desirable.

We also note the important fact that the wide amplitudes which are characteristic of the 11-year sun-spot curve, and which it has been chiefly sought to discover in the meteorological elements, does not, according to this research, appear at all prominently in the residuals. It is only the short period of about three years that displays the solar terrestrial synchronism. I am not, at present, able to indicate what this result implies in solar physics, but it certainly carries with it a change in our method of approaching the entire problem.

[20] The periodicity of sun spots and the variations of the mean annual temperatures of the atmosphere. M. Charles Nordmann. Comptes Rendus. Paris, June, 1903. Translation in Monthly Weather Review, August, 1903. P. 371.

[21] The simultaneous variations of sun spots and of terrestrial atmospheric temperatures. Prof. Alfred Angot. Annuaire de la Société Météorologique de France, June, 1903. Translation in Monthly Weather Review, August, 1903. P. 371.

III.—THE PROBLEM OF THE GENERAL CIRCULATION OF THE ATMOSPHERE OF THE EARTH.

THE CANAL THEORY.

In my Cloud Report, Annual Report of the Chief of the Weather Bureau, 1898–1899, Volume II, chapter 11, it was shown that for the United States the canal theory of the general circulation of the atmosphere, as worked out by Ferrel and by Oberbeck, does not sufficiently conform to the observations on cloud motions to be a satisfactory solution of the problem. The Report of the International Committee, 1903, by H. H. Hildebrandsson, reached the same conclusions for nearly all parts of the Northern Hemisphere, and, therefore, that canal theory may be finally abandoned. The following paper contains some suggestions on this subject which seem promising, and adapted to laying the foundation for a new development of this branch of theoretical meteorology. The physical facts to be accounted for may be found in the two publications referred to, also in my Papers on the Statics and Kinematics of the Atmosphere in the United States,[22] and they need not be recapitulated in this place.

THE GENERAL EQUATIONS OF MOTION.

Referring to the well-known general equations of motion as summarized in the Weather Bureau Cloud Report, from equation (155) we have

$$(1)\qquad -\frac{1}{\rho}\frac{\partial P}{\partial x}-\frac{\partial V}{\partial x}=\frac{du_1}{dt}.$$

$$-\frac{1}{\rho}\frac{\partial P}{\partial y}-\frac{\partial V}{\partial y}=\frac{dv_1}{dt}.$$

$$-\frac{1}{\rho}\frac{\partial P}{\partial z}-\frac{\partial V}{\partial z}=\frac{dw_1}{dt}.$$

These are transformed into the first form of polar equations (181), these again into the forms (200) and (201) in succession, so that the common integral becomes

$$(2)\qquad \int-\frac{dP}{\rho}=\int\left(\frac{du}{dt}\partial x+\frac{dv}{dt}\partial y+\frac{dw}{dt}\partial z\right)+V-C.$$

The usual method of development proceeds by taking

$$(3)\qquad u=\frac{\partial x}{\partial t},\quad v=\frac{\partial y}{\partial t},\quad w=\frac{\partial z}{\partial t},\qquad \text{so that}$$

$$(4)\qquad \int-\frac{dP}{\rho}=\int(udu+vdv+wdw)+V-C$$

$$=\tfrac{1}{2}(u^2+v^2+w^2)+V-C$$

$$=\tfrac{1}{2}q^2+V-C.$$

This is the ordinary form of the equation of motion on the rotating earth as given in treatises on hydrodynamics, as in Lamb, p. 22, and Basset, Vol. I, p. 34, and is known as Bernoulli's Theorem. C is not an absolute constant, but is the function of the parameter of a stream line; and in the atmosphere, where the flow takes place in stratified layers having different temperatures and angular momenta, it changes from one stratum to another.

It is also possible to integrate these terms along an arbitrary line, $s=\int ds=\int(dx, dy, dz)$, and in this case the derivative relative to the velocity will give acceleration along ds; that is, we have $\dot{q}ds$ instead of qdq, and under some circumstances this may prove to be an advantageous method. In meteorology this will depend, however, upon whether the one or the other set of terms that are required are most practically observed, as line integrals may be readily computed for either of these systems.

LINE INTEGRALS IN THE ATMOSPHERE.

The principles of the canal theory of circulation have been applied by V. Bjerknes[23] and J. W. Sandström[24] in their papers on circulation, under the form of line integrals around arbitrary closed curves in the atmosphere. Thus, the circulation is expressed by them, with the vertical and horizontal components of the total enclosed curve, as

	Total circulation.		Relative component.		Earth's component.
(5)	C_a	$=$	C	$+$	C_e

$$(6)\qquad \int q_a ds=\int q\,ds+2\,\omega_0 S_1$$

$$(7)\qquad \int\frac{dq_a}{dt}ds=\int\frac{dq}{dt}ds+2\,\omega_0\frac{dS_1}{dt}$$

$$(8)\qquad -\int\frac{dP}{\rho\,.\,ds}\cdot ds=\int\dot{q}\,ds+\frac{d}{dt}\cdot 2\,\omega_0\int\tfrac{1}{2}\varpi\cos i\cos\theta\,ds+R$$

$$(9)\qquad -\int\frac{dP}{\rho}=\int\dot{q}\,ds+2\,\omega_0\cos\theta\,.\,\frac{dS}{dt}+R$$

Equation (7) is the time rate of change.

C_a = the line integral of the tangential component of total velocity.

C = the line integral of the relative velocity (tangential).

C_e = the line integral of the velocity of a point on the moving earth itself (tangential).

(q_a, q, q_e) = the velocities; $(\dot{q}_a, \dot{q}, \dot{q}_e)$ = the accelerations.

R = friction; ω_0 = the angular velocity of the earth.

P = pressure; ρ = density.

i = the angle on the plane of the parallel of latitude that ds makes with the direction of a moving point of the earth.

S_1 = the projection of the closed curve S on the plane of the equator for the polar distance θ.

These integrations involve an accurate knowledge of the pressure, density, and acceleration at numerous points along the chosen closed curve, and this it is very difficult to obtain by practicable observations. The variation of S can be found more readily. Several illustrations are given by the authors in applying the theory to the general circulation of the atmosphere and to the local cyclones and anticyclones, but these illustrations do not seem to conform satisfactorily to the conditions observed in North America, as will be set forth in the other papers of this series and in a full report on the subject.

There arises no question with respect to any of the terms of the equation except the one containing $\frac{dS_1}{dt}$, which appears to be an addition to the usual form of the equation of motion on the rotating earth. As has been shown by V. Bjerknes, if the angle θ can be taken constant for a given relatively small closed curve, we have

$$(10)\qquad 2\,\omega_0\frac{dS_1}{dt}=2\,\omega_0\cos\theta\frac{d}{dt}\int\tfrac{1}{2}\varpi\cos i\,ds,$$

where i is the angle that the element ds makes with the parallel of latitude, or the angle between the two radii of an ele-

[22] Monthly Weather Review, Vol. XXX, pp. 13, 80, 117, 163, 250, 304, 347.

[23] Meteorol. Zeitschrift, March, 1900; April, 1900; November, 1900; March, 1902.

[24] Kon. Svens. Vet.—*Ak.* Handlingar, *Bd.* 83, No. 4; Meteorol. Zeitschrift, April, 1902; Vetens. *Ak.* 1902, No. 3.

mentary area, as shown in fig. 14. Hence, for a line integral we have,

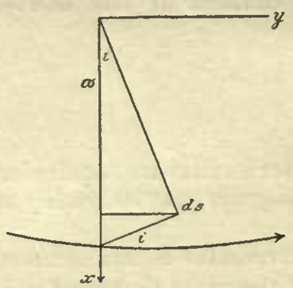


FIG. 14.—Component axes.

$$(11)\quad \frac{d}{dt}\int \tfrac{1}{2}\,\varpi \cos i.\, ds = \tfrac{1}{2}\int \frac{d\varpi}{dt}.\,\cos i\, ds - \tfrac{1}{2}\int \varpi \frac{di}{dt}.\,\sin i\, ds$$

$$= \tfrac{1}{2}\left(u\, dy - v\, dx\right),$$

since $\frac{d\varpi}{dt} = u, \quad \varpi\frac{di}{dt} = v, \quad ds \cos i = dy, \quad ds \sin i = dx.$

We have in the case of a velocity potential, $u\, dy - v\, dx = 0$; and, as is well known, the only influence of the rotation of the earth is to add a deflecting force always at right angles to the direction of motion. The integral of the work done in moving a particle, $\int \frac{dq}{dt}.\,ds$, receives no additional term from the fact that the earth rotates, any more than a planet alters the velocity in its orbit from a force perpendicular to its path. We thus obtain $2\,\omega_0 \frac{dS_1}{dt} = 0$, and all the developments derived from its use must be carefully interpreted. It seems important to have made this fact clear, in order that the equation used as the basis of the following analysis may be taken without modifications. If the gravity potential $V = gz$ is added we obtain the complete equation. The line integral of a gravity force around a closed curve is, also, always zero.

EQUIVALENT EXPRESSIONS FOR THE DENSITY ρ.

The specific volume or isoster, $\frac{1}{\rho} = v$, in the term $\frac{P}{\rho}$, can be discussed in four different ways, and substitutes for it can be introduced into the equation.

1. From Bigelow's equation (47*a*), Cloud Report, we have

$$(12)\qquad \frac{1}{\rho} = \frac{1}{\rho_0}\cdot\frac{P_0}{P}\,\frac{T}{T_0} = \frac{1}{\rho_0}\cdot\frac{P_0}{P}\,(1 + a\, t),$$

where the variations are expressed in terms of ρ_0, P_0, P and the thermometric temperature t. This is the common procedure among meteorologists.

2. From equation (75), the Boyle-Gay Lussac law of gases,

$$(13)\qquad \frac{1}{\rho} = \frac{R\,T}{p} = v,$$

where the variations are given in terms of R, T, p—the gas constant, the absolute temperature, and the weight—and this has been used in some discussions. Since the atmosphere is not arranged upon the adiabatic law, but diverges from it considerably, this method must be cautiously introduced, though there is a strong temptation to use the absolute temperature on account of its convenience.

3. Since we have $\frac{1}{\rho} = \left(\frac{p_0}{p}\right)^{\frac{1}{k}}\frac{1}{\rho_0}$, by equation (84), and $\frac{1}{\rho_0} = \frac{R\,T_0}{p_0}$, by (75), we obtain the third form,

$$(14)\qquad \frac{1}{\rho} = \left(\frac{p_0}{p}\right)^{\frac{1}{k}}\frac{R\,T_0}{p_0},$$

$$(15)\qquad \frac{1}{\rho} = p_0^{\frac{1-k}{k}}\,R\,T_0\,p^{-\frac{1}{k}},$$

where R is the gas constant, and $T_0 = \theta_0$ the potential temperature. This form was employed by H. von Helmholtz, and it has several advantages over the others in applications to the atmosphere.

4. By reducing the volume $\frac{1}{\rho}$ to unit density so that $\rho_0 = 1$, we shall find that

$$(16)\qquad \frac{1}{\rho} = \frac{k}{k-1}\,R^{\frac{1}{k}}\,\theta^{\frac{1}{k}}\,\frac{k-1}{k}\,p^{-\frac{1}{k}},$$

which is the form used by Emden in his paper on the solar circulation.

5. The potential temperature is found practically from the formula

$$(17)\qquad \theta = \theta_0\left(\frac{p}{p_0}\right)^{\frac{k-1}{k}} = \theta_0\left(\frac{B}{B_0}\right)^{0.2889},$$

or in logarithms,

$$(18)\qquad \log\theta = \log\theta_0 + 0.2889\,(\log B - \log B_0).$$

DEVELOPMENT OF THE TERMS $\frac{\omega}{\rho}$, V, AND $\frac{dr}{d\varpi}$.

Since the pressure P in units of force $= g_0 p$, we have from (15)

$$(19)\qquad \frac{P}{\rho} = g_0 p_0^{\frac{1-k}{k}}\,R.\theta.p^{-\frac{1}{k}}\,p = g_0 p_0^{\frac{1-k}{k}}\,R.\theta.p^{\frac{k-1}{k}}.$$

$$\left.\begin{array}{ll}(20) & \frac{P}{\rho} = A.\theta.\pi \\ (21) & \frac{\partial P}{\rho\partial\varpi} = A.\theta.\frac{\partial\pi}{\partial\varpi} \\ (22) & \frac{\partial P}{\rho\partial r} = A.\theta.\frac{\partial\pi}{\partial r}\end{array}\right\}\ \text{for}\ \left\{\begin{array}{l} A = g_0 p_0^{\frac{1-k}{k}}\,R. = \text{constant.} \\ \theta = \theta_0\left(\frac{p}{p_0}\right)^{\frac{1-k}{k}}. \\ \pi = p^{\frac{k-1}{k}} = p^{0.2889}.\end{array}\right.$$

The gravity potential, including the centrifugal force of rotation about the axis z, with the angular velocity ω_0, at the distance ϖ is, for the positive direction of r outwards,

$$(23)\qquad -V = +gr - \tfrac{1}{2}\,\omega_0^2\,\varpi^2.$$

$$(24)\qquad -V = \frac{g_0 R^2}{r} - \tfrac{1}{2}\,v_0^2.$$

Hence the original equation (4) is transformed as follows:

$$(25)\qquad \frac{P}{\rho} = -\tfrac{1}{2}\,(u^2 + v^2 + w^2) - V + C.$$

$$(26)\qquad A\theta\pi = -\tfrac{1}{2}\,(u^2 + v^2 + w^2) - \tfrac{1}{2}\,v_0^2 + \frac{g_0 R^2}{r} + C.$$

$$(27)\qquad A\theta\pi = -\tfrac{1}{2}\,(v^2 + v_0^2) - \tfrac{1}{2}\,(u^2 + w^2) + \frac{g_0 R^2}{r} + C.$$

The equations of motion for two strata flowing over each other, and having different potential temperatures and angular momenta, become,

(28) First stratum:

$$\frac{1}{\theta_1}\frac{g_0 R^2}{r} = A\pi_1 + \tfrac{1}{2}\,(v_1^2 + v_0^2)\,\frac{1}{\theta_1} - \frac{C_1}{\theta_1} + \tfrac{1}{2}\,(u^2 + w^2)_1\,\frac{1}{\theta_1}.$$

(29) Second stratum:

$$\frac{1}{\theta^2}\frac{g_0 R^2}{r} = A\pi_2 + \tfrac{1}{2}\,(v_2^2 + v_0^2)\,\frac{1}{\theta_2} - \frac{C_2}{\theta_2} + \tfrac{1}{2}\,(u^2 + w^2)_2\,\frac{1}{\theta_2}.$$

At the discontinuous surface of flow the pressure $\pi_1 = \pi_2$, hence,

$$(30)\qquad \left(\frac{1}{\theta_1} - \frac{1}{\theta_2}\right)\frac{g_0 R^2}{r} = \tfrac{1}{2}\,\frac{(v_1^2 + v_0^2)}{\theta_1} - \tfrac{1}{2}\,\frac{(v_2^2 + v_0^2)}{\theta_2}$$

$$-\frac{C_1}{\theta_1} + \frac{C_2}{\theta_2} + \tfrac{1}{2}\,\frac{(u^2 + w^2)_1}{\theta_1} - \tfrac{1}{2}\,\frac{(u^2 + w^2)_2}{\theta_2}.$$

The terms in u and w may not always be neglected where there are strong meridional and vertical currents, as in cyclones and anticyclones.

TO FIND THE DIRECTION OF THE BOUNDARY CURVE BETWEEN TWO STRATA.

1. Differentiate (27) for r with ϖ constant.

$$(31)\qquad A\theta d\pi = -\frac{g_0 R^2 dr}{r^2} = -g dr.$$

Then, in crossing the boundary from the first to the second stratum,

$$(32)\qquad A\frac{d(\pi_1-\pi_2)}{dr} = -g\left(\frac{1}{\theta_1}-\frac{1}{\theta_2}\right) = -g\left[\frac{\theta_2-\theta_1}{\theta_1\theta_2}\right].$$

2. Differentiate for ϖ with r constant, at the same time holding the angular momentum ($v\varpi$) constant in each stratum. Equation (27) can be written:

$$(33)\qquad g_0\frac{R^2}{r} = A\theta\pi + \frac{1}{2}\frac{(v^2\varpi^2)}{\varpi^2} + \frac{1}{2}\omega_0^2\varpi^2 + \frac{1}{2}(u^2+w^2) - C.$$

Differentiating,

$$(34)\qquad 0 = A\theta d\pi - \frac{1}{2}\cdot\frac{2\varpi(v^2\varpi^2)d\varpi}{\varpi^4} + \frac{1}{2}\cdot 2\omega_0^2\varpi d\varpi + u\frac{du}{d\varpi} + \frac{wdw}{d\varpi}.$$

$$(35)\qquad A\theta d\pi = +v^2\frac{d\varpi}{\varpi} - v_0^2\frac{d\varpi}{\varpi} - \left(\frac{udu}{d\varpi}+\frac{wdw}{d\varpi}\right).$$

For the two strata,

$$(36)\qquad A\frac{d(\pi_1-\pi_2)}{d\varpi} = \frac{1}{\varpi}\left(\frac{v_1^2-v_0^2}{\theta_1} - \frac{v_2^2-v_0^2}{\theta_2}\right)$$

$$-\frac{1}{d\varpi}\left[\left(\frac{udu}{d\varpi}+\frac{wdw}{d\varpi}\right)_1\frac{1}{\theta_1} - \left(\frac{udu}{d\varpi}+\frac{wdw}{d\varpi}\right)_2\frac{1}{\theta_2}\right]$$

$$= \frac{1}{\varpi}\left[\frac{(v_1^2-v_0^2)\,\theta_2-(v_2^2-v_0^2)\,\theta_1}{\theta_1\theta_2}\right],$$

omitting terms of the second order.

3. Finally, dividing (36) by (32), we obtain,

$$(37)\qquad \frac{dr}{d\varpi} = -\frac{1}{g\varpi}\left[\frac{(v_1^2-v_0^2)\,\theta_2-(v_2^2-v_0^2)\,\theta_1}{\theta_2-\theta_1}\right].$$

This equation defines the slope of the curve which separates the two stratified currents that flow past each other, preserving their angular momenta, $\Omega = v\varpi = \omega\varpi^2 = constant$, according to the vortex law, where ω is the total angular velocity upon the rotating earth and ϖ is the distance from the axis of rotation. It can be written and interpreted in three different ways, and this gives rise to three cases, each of which finds its application in atmospheric circulations. The equations given in the papers by von Helmholtz and by Emden can be readily transposed into Case I and Case III, but Case II has not been considered heretofore. Omitting terms in u and w, these three cases may be expressed as in equations (38), (39), and (40), following.

CASE I. APPLICABLE TO THE TEMPERATE AND POLAR LATITUDES OF THE EARTH.

$$\theta_1 > \theta_2 \text{ and } \frac{v_1^2-v_0^2}{\theta_1} > \frac{v_2^2-v_0^2}{\theta_2} \text{ for } \begin{bmatrix} v_1 > v_0 \\ v_2 > v_0 \\ v_1 > v_2 \end{bmatrix} \begin{matrix}\text{eastward}\\ \text{relative}\\ \text{velocities.}\end{matrix}$$

$$(38)\qquad \frac{+dr}{-d\varpi} = -\left[\frac{(v_2^2-v_0^2)\,\theta_1-(v_1^2-v_0^2)\,\theta_2}{\theta_1-\theta_2}\right] = -\left[\frac{-}{+}\right].$$

The second member of the equation is positive if

$$\frac{v_1^2-v_0^2}{\theta_1} > \frac{v_2^2-v_0^2}{\theta_2},$$

where $v_1 > v_0$, $v_2 > v_0$, $v_1 > v_2$, and $\theta_1 > \theta_2$, that is to say, if the higher strata have a higher potential temperature and greater eastward relative velocity than the lower, the quantities being arranged as in fig. 15.

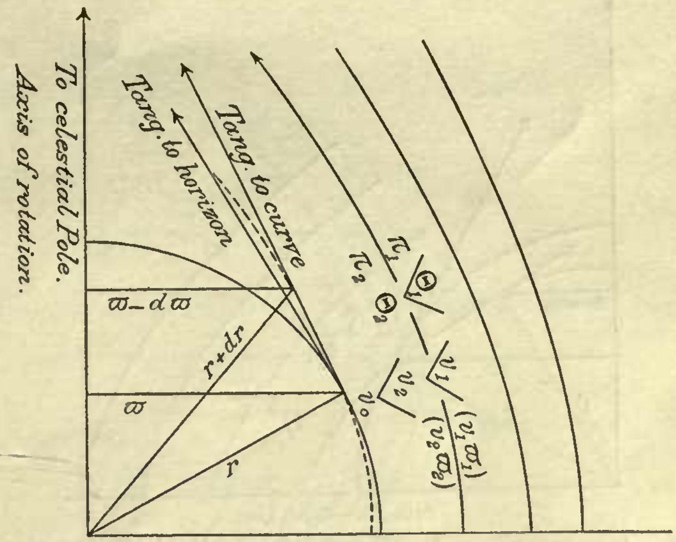


FIG. 15.—Case I.

Take a point in the atmosphere defined by (r, ϖ) the radius and the radius of rotation, respectively. The next successive point on the line of separation of the two gyrating strata is given by ($r+dr$), ($\varpi - d\varpi$) as indicated, so that the curve continually rises above the successive tangents to the horizon, but approaches the axis of rotation in the direction of the celestial pole. Since ($v_1^2 - v_0^2$) is the square of the relative linear eastward velocity, it follows that the strata in the atmosphere subject to this law have a continually greater eastward drift and greater potential temperatures with the increase in altitude above the surface. These conditions are characteristic of the earth's atmosphere beyond a certain latitude which varies with the height above the surface. The Weather Bureau Cloud Report, 1898, proved that the velocities and also the potential temperatures for the United States conform to Case I, as in chapters 12, 13, and 14, which contain a discussion of the departure of the temperatures of the upper strata from the adiabatic law in the sense that these strata are overheated. Those velocities have been properly prepared for immediate introduction into the above formula.

CASE II. APPLICABLE TO THE TROPICAL ZONES OF THE EARTH.

$$\theta_1 < \theta_2 \text{ and } \frac{v_1^2-v_0^2}{\theta_1} < \frac{v_2^2-v_0^2}{\theta_2} \text{ for } \begin{bmatrix} v_1 < v_0 \\ v_2 < v_0 \\ v_1 > v_2 \end{bmatrix} \begin{matrix}\text{westward}\\ \text{relative}\\ \text{velocities.}\end{matrix}$$

$$(39)\qquad \frac{-dr}{-d\varpi} = -\frac{1}{g\varpi}\left[\frac{(v_2^2-v_0^2)\,\theta_1-(v_1^2-v_0^2)\,\theta_2}{\theta_1-\theta_2}\right] = -\left[\frac{-}{-}\right]$$

The second member of the equation is negative if

$\dfrac{v_2^2-v_0^2}{\theta_2} > \dfrac{v_1^2-v_0^2}{\theta_1}$, where $v_1 < v_0$, $v_2 < v_0$, $v_1 > v_2$, and $\theta_1 < \theta_2$, that is to say, if the higher strata have lower potential temperatures than the lower, and the lower strata a greater westward relative velocity than the higher, the quantities being arranged as in fig. 16.

Take a point in the atmosphere defined by (r, ϖ) and the next successive point on the line of separation is given by ($r-dr$), ($\varpi - d\varpi$), as indicated, so that the curve continually falls below the successive tangents to the horizon, and approaches the axis of rotation in the direction of the celestial pole. The relative velocity is westward, since v_0 is greater than v_1 and v_2, so that $v_1^2 - v_0^2$ and $v_2^2 - v_0^2$ are both negative quantities. Since $v_1^2 - v_0^2$ is a smaller negative quantity than $v_2^2 - v_0^2$, the numerator is negative. Also, the denominator is negative, for $\theta_1 < \theta_2$. These conditions are fulfilled in the tropical zones where the westward drift is greater in the lower strata and diminishes upward, while the potential tempera-

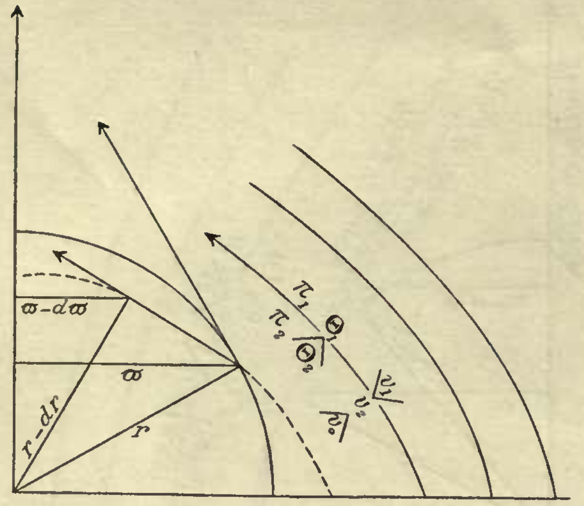


FIG. 16.—Case II.

tures decrease upward. Chapter 8 of the full report will discuss the velocities in the tropical zones of the West Indies. The potential temperatures in the Tropics still remain to be computed.

CASE III. APPLICABLE TO THE ATMOSPHERES OF THE SUN, JUPITER, AND SATURN.

$$\theta_1 > \theta_2 \text{ and } \frac{v_1^2 - v_0^2}{\theta_1} < \frac{v_2^2 - v_0^2}{\theta_2} \text{ for } \begin{bmatrix} v_1 > v_0 \\ v_2 > v_0 \\ v_1 < v_2 \end{bmatrix} \begin{array}{l} \text{eastward} \\ \text{relative} \\ \text{velocities.} \end{array}$$

$$(40) \quad \frac{+dr}{+d\varpi} = -\frac{1}{g\varpi}\left[\frac{(v_2^2 - v_0^2)\,\theta_1 - (v_1^2 - v_0^2)\,\theta_2}{\theta_1 - \theta_2}\right] = -\left[\frac{+}{+}\right]$$

The second member of the equation is negative if

$$\frac{v_2^2 - v_0^2}{\theta_2} > \frac{v_1^2 - v_0^2}{\theta_1}, \text{ where } v_1 > v_0,\ v_2 > v_0, v_1 < v_2, \text{ and } \theta_1 > \theta_2,$$

that is to say, if the higher strata have a higher potential temperature and a smaller eastward relative velocity than the lower, the quantities being arranged as in fig. 17.

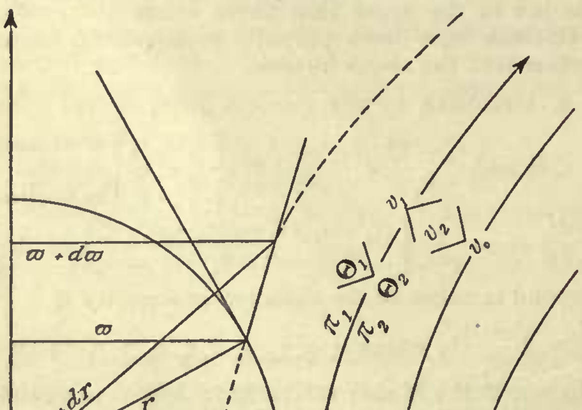


FIG. 17.—Case III.

Take a point in the atmosphere defined by (r, ϖ), and the next successive point on the line of separation, which has varying temperatures but angular momenta that are constant within the thin layers, is given by $(r+dr)$ $(\varpi+d\varpi)$, as indicated, so that the curve continually rises above the plane of the horizon, and recedes from the axis of rotation in the direction of the celestial pole. The warmer strata are nearer the axis, and the potential temperature increases in the direction parallel to the axis of rotation, and at the same time the relative velocity is such that the strata near the pole rotate more slowly than those at greater distances. These conditions are found to prevail in the atmospheres of the sun, also of the planets Jupiter and Saturn, as attested by the belt formations and the systems of vortices penetrating to the surface. On the sun the granules of the photosphere are the ends of vortex tubes between adjacent strata having different velocities. Similar vortex tubes are seen on the two planets.

THE INTERACTION OF CASE I AND CASE II IN THE EARTH'S ATMOSPHERE IN THE FORMATION OF LOCAL CYCLONES AND ANTICYCLONES.

In the earth's atmosphere the boundary between the eastward drift of the temperate zones and the westward drift of the tropical zones is an arch spanning the equator high up into the cirrus cloud strata, and resting on the surface at latitudes 30° to 25°. On the poleward side Case I applies but on the side toward the equator Case II prevails.

If the circulations of Case I in the temperate and polar zones, and of Case II in the tropical zones, are applied without further conditions, the isobars in the atmosphere will be distributed, as in fig. 18, so that they rise from the arched boundary of the eastward and the westward relative velocities toward the pole and toward the plane of the equator respectively. This, however, is not the course of the surfaces of pressure in the atmosphere as determined by the observations near sea level, and by computations at higher levels. To illustrate the actual conditions, in fig. 20 Ferrel's values of the isobars on the sea level are given from pole to pole, and Sprung's isobars for the 2000-meter and the 4000-meter planes. The practical problem is, therefore, to account satisfactorily for the modifications of the types. In the present state of meteorology we enter upon a field that is incompletely explored, so that the following remarks are suggestive of the solution rather than final, but there will be much material that sustains them in the complete report, Volume II, Report of the Chief of the Weather Bureau, 1903–1904.

There are two conditions that modify the solutions of Case I and Case II very decisively. (1) The first is that the assumption that the angular momenta in the several strata remain constant around the earth, or that the air rotates in unbroken rings, does not hold good even approximately. Besides the waves and vortices engendered between discontinuous strata, as von Helmholtz explained, there is a yet more powful cause for the breaking down of the vortex law, $v\,\varpi =$ constant, namely, in the cyclones and the anticyclones of middle latitudes, and in the convectional vertical circulation near the equator. (2) The second is that the boundary between the eastward and the westward drift does not girdle the earth uniformly, but is broken up into sections by the intrusion of Case II into the region of Case I, and the extension of Case I into the region of Case II, so that the high pressure belt which this solution assumes to encircle the earth is broken up into large isolated high areas or centers of action, as those lying over the oceans in summer, or over the continents in winter, in the lower strata of the atmosphere. To work out the theory of these details will be a large task for the meteorologist of the future. These two types of disturbance operate together, somewhat as described in the Weather Bureau Cloud Report, 1898–1899, so that the present paper is merely an extension of the analysis there suggested. The following descriptive statement attempts to outline the probable course of the modifications of the pure vortex theory contained in the system of equations given above.

Referring to figs. 18 and 19, the "unmodified" and the "modified" systems, respectively, it is evident that the solar radiation in the Tropics, if unrelieved, will by accumulation raise the isobars of Case II, by increasing the potential temperature θ_2 and the westward velocity $v_2 - v_0$ in the lower strata. In a circulating atmosphere the relief comes in two ways, (1) by forming a vertical convection near the equator, and (2) by forcing a horizontal convection into the lower strata

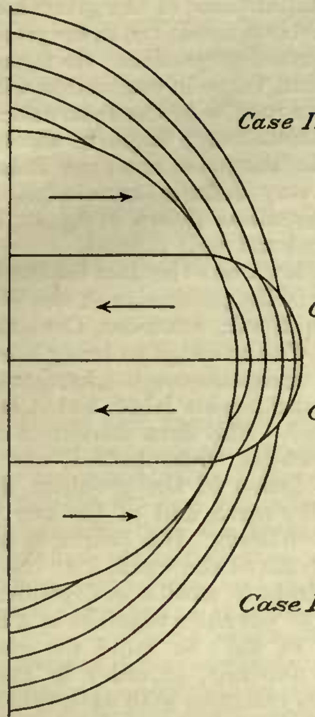


FIG. 18.—Cases I and II unmodified.

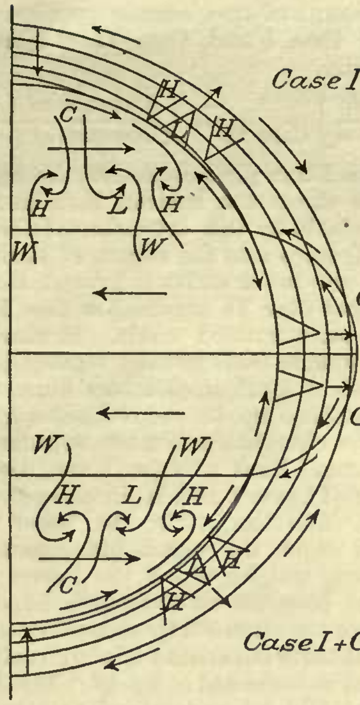


FIG. 19.—Cases I and II as modified.

of the temperate zones. The first transports heat into the upper strata, reducing θ_2 and increasing θ_1, so that the westward drift diminishes. At the same time the intrusion of masses of air having one value of momentum $(mv)_{II}$ into those having another value $(mv)_I$ will change their velocities. These two causes lower the lines of Case II on the equator side, and in the lower strata may even reverse them. Accompanying these changes a component on the meridian toward the equator sets in, so that the trades from the northeast and southeast are developed, and the first minor circulation is maintained in the sense indicated by the arrows over the tropical zone of fig. 19. The rise and fall of the isobars of Case II, with the relief of the incoming solar heat through this circulation, is a complex but sensitive form of natural heat governor which is self-regulating, and preserves the normal state of equilibrium proper for the season of the year. This special action is chiefly due to the mutual movement among the terms of equation (39) for Case II.

A still more complex system relates to the temperate zones and Case I. To some extent the terms within equation (38) for Case I go through a similar self-adjustment in response to the local insolation, but this is by no means the primary cause for the depression of the isobars of fig. 18 to those of fig. 19. As explained in my paper, "The mechanism of countercurrents of different temperatures in cyclones and anticyclones," MONTHLY WEATHER REVIEW, February, 1903, cyclones and anticyclones are formed by horizontal currents underflowing the prevailing eastward drift. Thus, as shown on fig. 19, warm currents flow from the Tropics into the Temperate Zone, as from the Gulf of Mexico into the United States, underneath the eastward drift, and this stratification of warm air beneath cold air produces two changes. The potential temperature θ_2 is increased, the value $\theta_1 - \theta_2$ is diminished, the velocity is checked and the isobars fall, because the angular momentum is diminished. At the same time that the air rises on the east side of the cyclone, a cold current from the north flows to the west side, and this decreases its θ_2 but increases the difference $\theta_1 - \theta_2$, so that the velocities are increased. It is known that the eastern warm current tends to curl westward and the western cold current tends to curl eastward about a cyclonic center; inverted conditions prevail around an anticyclonic center. Furthermore, the dynamic action of intruding cyclones and anticyclones from the lower to the higher strata, by their interchange of inertia with the eastward drift, must diminish the eastward velocity and lower the isobars of Case I.

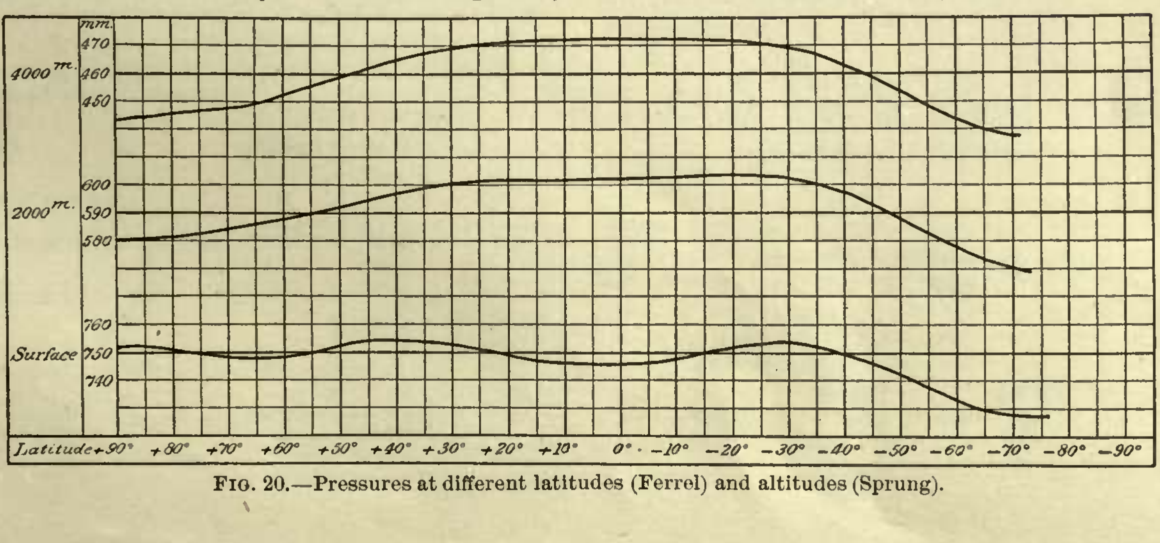


FIG. 20.—Pressures at different latitudes (Ferrel) and altitudes (Sprung).

This effect of the interchange of components may be seen by combining the terms of Case I and Case II algebraically. Thus, we have, symbolically,

$$\left[\frac{+dr}{-d\varpi}\right]_{\text{Case I}} + \left[\frac{-dr}{-d\varpi}\right]_{\text{Case II}} = \left[\frac{\text{decrease of }(+dr)}{\text{increase of }(-d\varpi)}\right]$$

so that the lines of Case I are plotted nearer the axis, and lower in the atmosphere above the horizon than in fig. 18. There are instances in which, by this intrusion of the warm air of Case II from the Tropics into the region of Case I, the potential temperature of the lower strata is greater than that of the higher strata, so that Case II supersedes Case I in the temperate zones with local westward winds. Similarly, the interplay of these cases outside their normal regions is a sufficient cause for the manifold local circulations found in the lower strata of the atmosphere up to about 3 miles from the ground, beyond which the circulation is more regular. The amount by which the normal lines of Case I are depressed through the intermixture of Cases I and II, in consequence of temperature and inertia interchanges in the lower strata, measures the amount by which the vortex law ceases to be complete in its application, and by which the Ferrel theory of the general circulation becomes an untenable hypothesis. In effect these interchanges are attended by secondary currents along the meridian so that there is a second minor circuit in the temperate zones, somewhat as indicated on fig. 19. The *H, L, H,* of the vertical section should be understood to stand over *H, L, H,* on the horizontal plane of the given latitude; that is, they are not distributed in latitude but in longitude, and should be superposed in a correct projection. So far as I understand the facts, this circulation, taken in connection with the tropical circuit, conforms to the results of the International Survey, as stated in H. H. Hildebrandson's Report, which need not be here recapitulated. In the polar zone our information is too meager to afford us very definite knowledge, but I suspect that there is a third circuit as shown in fig. 19, though it may not be very pronounced and well defined.

It is my purpose to work out the data for the temperate and the tropical zones now in the possession of the Weather Bureau and applicable to the North American Continent, along the lines here indicated. The attempt to bring these laws of the general and the local circulations into a harmonious numerical scheme will require considerable labor, but it is believed that it can be accomplished. The data contained in my reports, while apparently somewhat disconnected, are in reality all contributory to my solution of the problems of atmospheric circulations both of the earth and of the sun, together with the connections between them. It is proper to determine carefully the separate portions of the work, i. e., the velocities and temperatures of the strata in motion as dependent upon observations, before trying to put them together in a final synthesis. It is only necessary to have in mind the general plan of development, as here outlined, in order to keep the several portions in harmonious relations with each other.

IV.—VALUES OF CERTAIN METEOROLOGICAL QUANTITIES FOR THE SUN.

THE IMPORTANCE OF THESE VALUES TO TERRESTRIAL METEOROLOGY.

The most important data needed for use in studies in solar physics are the correct values of the pressure, the temperature, the density, the gas constant, and their many derived relations, at the surface of the sun, within its mass, and throughout the gaseous envelope. In the present uncertain state of our knowledge of these quantities, even an approximate derivation of these data is important, and this forms the justification for the studies contained in this paper. The problems of the circulation within the sun's photosphere, the transitions and the transformations in the atmospheric envelope with the attendant radiations and absorptions, the heat and light received at the outer surface of the earth's atmosphere, the resulting absorption and transmission of energy in the air, and the dependent circulation, are all languishing for the lack of a sound footing for our computations and deductions. The computations for the surface temperature of the sun give results ranging from 5000° to 10,000°; using Ritter's Law, Professor Schuster computes the temperature at the center of the sun as 12,000,000°, assuming that it is composed of hydrogen split up into monatomic elements. But it is evident that any such range of temperature would simply explode the sun, whereas it now circulates in a moderate manner. Unless some value for the temperature of the solar photosphere can be found, it will be impossible to determine what percentage of the total solar radiation is absorbed in the solar envelope, even though the radiant heat be computed successfully on the outer surface of the earth's atmosphere from radiation measurements at the ground. Should the following remarks prove to be merely suggestive it will be proper to make them as a contribution to the problems in solar physics.

I have been interested in the paper by Prof. F. E. Nipher, on the "Law of contraction of gaseous nebulæ,"[25] because it seems to offer a way of escape from the impossible results which follow from Ritter's equations, where the exponent in $P\,v^n = B$ is 1.33 +. Nipher makes the value of $n = 1.10$, and from this exponent the entire system of relations seems to be more probable. I will recapitulate Nipher's equations, after making the following changes in his notation to reduce them to the symbols used in my papers:

	Nipher.		Bigelow.
Gas constant	change C	to	R
Density	" δ	"	ρ
Distance from center	" R	"	r
Mechanical equivalent of heat	" J	"	$A' = \dfrac{1}{A}$
Heat equivalent of work	" $\dfrac{1}{J}$	"	$A = \dfrac{1}{A'}$
Constant	" A	"	B
Ratio	" ρ	"	b
Constant	" k	"	k^2

[25] Transactions Academy of Science, St. Louis, October 1, 1903.

NIPHER'S EQUATIONS.

Adiabatic law for perfect gases:

$$(41)\qquad P\,v = R\,T.$$

Heat relation:

$$(42)\qquad dQ = c_v dT + P\,dv.$$

Assumed laws for non-perfect gases:

$$(43)\qquad P\,v^n = P_0 v_0^{\,n} = B.$$

$$(44)\qquad T\,v^{n-1} = \frac{B}{R}.$$

$$(45)\qquad \frac{T^n}{P^{n-1}} = \frac{B}{R^n}.$$

Specific heat:

$$(46)\qquad \left(\frac{dQ}{dT}\right)_n = c_v + \frac{AR}{1-n} \qquad\qquad A = \frac{1}{4.19\times 10^7}.$$

Gravitation:

$$(47)\qquad \frac{dP}{dr} = -k^2\frac{M}{r^2}\rho = -k^2\frac{M}{r^2}\left(\frac{P}{B}\right)^{\frac{1}{n}} \qquad k^2 = \frac{1}{1.5173\times 10^7}.$$

Pressure:

$$(48)\qquad P = \left[\frac{4n-3n^2}{(2-n)^2}\cdot\frac{B^{\frac{2}{n}}}{2\pi\,k^2\,r^2}\right]^{\frac{n}{2-n}} = \left[\frac{0.95B^{1.82}}{2\pi\,k^2\,r^2}\right]^{1.22}$$

$$= \frac{0.95R^2\,T^2}{2\pi\,k^2\,r^2} = \frac{0.636}{8\pi}\,\frac{M^2\,k^2}{r^4}.$$

Density:

$$(49)\qquad \rho = \left[\frac{4n-3n^2}{(2-n)^2}\cdot\frac{B}{2\pi\,k^2\,r^2}\right]^{\frac{1}{2-n}} = \left[\frac{0.95B}{2\pi\,k^2\,r^2}\right]^{1.11}$$

$$= 0.95\,\frac{R\,T}{2\pi\,k^2\,r^2} = \frac{0.78}{4\pi}\,\frac{M}{r^3}.$$

Temperature:

$$(50)\qquad T = \frac{B^{\frac{1}{2-n}}}{R}\left[\frac{4n-3n^2}{(2-n)^2}\cdot\frac{1}{2\pi\,k^2\,r^2}\right]^{\frac{n-1}{2-n}} = \frac{B^{1.11}}{R}\left(\frac{0.95}{2\pi\,k^2\,r^2}\right)^{0.111}$$

$$= 0.818\,\frac{M\,k^2}{2\,R\,r}.$$

Mass:

$$(51)\qquad M = 4\pi\left(\frac{2-n}{4-3n}\right)\left[\frac{B(4n-3n^2)}{2\pi\,k^2(2-n)^2}\right]^{\frac{1}{2-n}} r^{\frac{4-3n}{2-n}}$$

$$= 5.14\pi\left(\frac{0.95B}{2\pi\,k^2}\right)^{1.11} r^{0.77}$$

$$= \frac{n}{2-n}\cdot\frac{2r\,R\,T}{k^2} = 1.22\,\frac{2R\,T\,r}{k^2}.$$

Weight of one gram at the surface:

$$(52)\quad g=\frac{k^2\,M}{r^2}=4\pi\,k^2\left(\frac{2-n}{4-3n}\right)\left[\frac{B(4n-3n^2)}{2\pi\,k^2(2-n)^2}\right]^{\frac{1}{2-n}}\frac{1}{r^{\frac{n}{2-n}}}$$

$$=5.14\pi\,k^2\left(\frac{0.95B}{2\pi\,k^2}\right)^{1.11}\ r^{\frac{1}{1.22}}$$

$$=\frac{n}{2-n}\cdot\frac{2R\,T}{r}=1.22\,\frac{2R\,T}{r}.$$

Auxiliaries:

$$(53)\quad B=P\,v^n=\frac{k^2}{2n}\left(\frac{4\pi}{4-3n}\right)^{\frac{1}{3}}(2-n)^{\frac{4}{3}}\,M^{\frac{2}{3}}$$

$$=\left[\frac{2\pi\,k^2}{(4-3n)n}\right]^{\frac{1}{3}}\left[(2-n)R\,T\,r\right]^{\frac{2}{3}}.$$

$$(54)\quad B'=\left[\frac{(4n-3n^2)B}{(2-n)^2 2\pi k^2}\right]^{\frac{1}{2-n}}.$$

Contraction ratio $b=\frac{r_0}{r}\frac{\text{(initial)}}{\text{(final)}}$:

$$(55)\quad P=P_0\left(\frac{r_0}{r}\right)^4=P_0\,b^4.$$

$$(56)\quad \rho=\rho_0\left(\frac{r_0}{r}\right)^3=\rho_0\,b^3.$$

$$(57)\quad T=T_0\left(\frac{r}{r_0}\right)=T_0\,b^{-1}.$$

Mass:

$$(58)\quad M=\frac{4}{3}\pi\,r^3\cdot\rho_a=4\pi\,\frac{2-n}{4-3n}\rho\cdot r^3.$$

Average density:

$$(59)\quad \rho_a=3\,\frac{2-n}{4-3n}\,\frac{B'}{r^{\frac{2}{2-n}}}=3\,\frac{2-n}{4-3n}\,\rho=\ldots\ldots\ldots\ldots3.86\,\rho.$$

Distance from center to stratum where the density ρ = average density ρ_a:

$$(60)\quad r_a=\left[\frac{4-3n}{3\,(2-n)}\right]^{\frac{2-n}{2}}r=\ldots\ldots\ldots\ldots\ldots\ldots0.545\,r.$$

Average pressure:

$$(61)\quad P_a=\frac{4\pi\int_0^r r^2\,P\,dr}{4\pi\int_0^r r^2\,dr}=3\,\frac{2-n}{6-5n}\,P=\ldots\ldots\ldots\ldots540\,P.$$

Distance from center to stratum where the pressure P = average pressure P_a:

$$(62)\quad r_a=\left[\frac{1}{3}\cdot\frac{6-5n}{2-n}\right]^{\frac{2-n}{2n}}r=\ldots\ldots\ldots\ldots\ldots\ldots0.502\,r.$$

Average temperature:

$$(63)\quad T_a=3\,.\,\frac{2-n}{8-5n}\,T=\ldots\ldots\ldots\ldots\ldots\ldots1.08\,T.$$

Distance from center to stratum where the temperature T = average temperature T_a:

$$(64)\quad r_a=\left[\frac{1}{3}\cdot\frac{8-5n}{2-n}\right]^{\frac{2-n}{2(n-1)}}r=\ldots\ldots\ldots\ldots\ldots\ldots0.707\,r.$$

Specific heat:

$$(65)\quad \frac{dQ}{dT}=\frac{c_v\frac{dP}{P}+c_p\frac{dv}{v}}{\frac{dP}{P}+\frac{dv}{v}}=\frac{c_p-n\,c_v}{1-n}=\frac{c_p}{\kappa}\left(\frac{\kappa-n}{1-n}\right).$$

Auxiliaries.

$$(66)\quad \left(\frac{dQ}{dT}\right)_n=c_p-A\,T\left(\frac{dv}{dT}\right)_p\left(\frac{dP}{dT}\right)_n \qquad \frac{dP}{P}=-n\,\frac{dv}{v}.$$

$$=c_p-A\,T\,\frac{R\,M}{P}\cdot\frac{R\,M}{v} \qquad v^n\,dP=-n\,P\,v^{n-1}dv.$$

$$=c_p-A\,R\,M^2 \qquad \frac{dP}{P}=-\frac{2n}{2-n}\,\frac{dr}{r}.$$

$$=c_p+4A\,R \qquad \frac{dv}{v}=+\frac{2}{2-n}\,\frac{dr}{r}.$$

$$=c_v+\frac{A\,R}{1-n}=-7.365\text{ (reversing sign).}$$

$$(67)\quad n=\frac{2c_p+4A\,R}{2c_p+3A\,R}=\frac{6\kappa-4}{5\kappa-3}=1.10.$$

$$(68)\quad c_p=A\,R\,\frac{3n-4}{2-2n}=A\,R\,\frac{\kappa}{\kappa-1}=A\,R\,\frac{4-3c}{2c-1}.$$

$$(69)\quad R\,T=P\,v=\frac{4}{3}\pi\,r^3\,P=\frac{2-n}{n}\cdot\frac{M\,k^2}{2r}=0.818\,\frac{M\,k^2}{2r}.$$

Heat:

$$(70)\quad Q=\left(\frac{dQ}{dT}\right)_n(T-T_0)=-(c_p+4A\,R)\,(T-T_0)$$
$$=-(c_p+4A\,R)\,T_0\,(b-1).$$

Work:

$$(71)\quad W=\int P\,dv=P_0\,v_0^n\int_{v_0}^{v}\frac{dv}{v^n}=\frac{P_0\,v_0}{1-n}(b-1)$$

$$=\frac{A\,R\,T_0}{1-n}(b-1)=-(c_v+c_p+4A\,R)T_0(b-1)$$

$$=4\pi\int_r^\infty r^2\,P\,dr=\frac{4-3n}{n}\,\frac{M^2\,k^2}{2r}=0.636\,\frac{M^2\,k^2}{2r}.$$

Ratios:

$$(72)\quad c=\frac{Q}{W}=\frac{c_p+4A\,R}{c_v+c_p+4A\,R}=\frac{c_p+4A\,R}{2c_p+3A\,R}=\frac{5\kappa-4}{5\kappa-3}=0.75.$$

$$(73)\quad \frac{Q}{W-Q}=\frac{c_p+4A\,R}{c_p-A\,R}=\frac{c_p+4A\,R}{c_v}=\ldots\ldots\ldots3.00.$$

$$(74)\quad \frac{W}{W-Q}=5\kappa-3=\ldots\ldots\ldots\ldots\ldots\ldots\ldots4.00.$$

Differences:

$$(75)\quad c_p-c_v=\frac{8-5n}{3(2-n)^2}\cdot\frac{4-3n}{5\kappa-3}\cdot A\,R=\ldots\ldots\ 0.180A\,R$$

$$=\frac{1}{5\kappa-3}\cdot\frac{4-3n}{n}\cdot\frac{M\,k^2\,A}{2r\ \,T_a}.$$

For a rise of 1° C., energy equivalent to $2c_p+3A\,R$ heat units must be applied to the unit mass, of which $c_p+4A\,R$ heat units are radiated per unit time, and $c_p-A\,R=c_v$ heat units are used in raising the temperature.

THE ASTRONOMICAL CONSTANTS FOR THE EARTH AND THE SUN.

It is difficult to select from the available astronomical data a system of constants that is rigorously self-consistent, and in this preliminary discussion it is not necessary to make complete adjustments between the several quantities. The fundamental units employed are conveniently the C. G. S. system, and not the C. S. system, because in the thermodynamic formulæ the unit of mass is the gram. In the C. G. S. system the gravitation constant is found from the formula,

$$(76)\quad g_0=k^2\,\frac{M_1\,m}{R_1^{\,2}},\text{ so that, }k^2=\frac{R_1^{\,2}\,g_0}{M_1\,m}.$$

The constant for transformations from the C. S. system to the C. G. S. system is $\frac{1}{k^2}$; i. e., (mass C. S.) $\frac{1}{k^2}$ = (mass C. G. S.).

TABLE 7.—*Astronomical constants.*

	Numbers.	Logarithms.
R_1 = mean radius of earth, *Bessel's* spheroid	6370 19100 cm.	8.8041525
$R_1^2 =$		17.6083050
$R_1^3 =$		26.4124575
ρ_{a1} = average density of earth, *Harkness*	5.576	0.746323
$\frac{4}{3}\pi =$	4.1888	0.622089
$M_1 = \frac{4}{3}\pi R_1^3 \rho_{a1}$ = mass of the earth in grams	6.0377×180^{27}	27.78070
m = 1 gram	1.00	0.000000
g_o = acceleration per second at surface of earth	980.60 cm.	2.991492
$k^2 = \frac{R_1^2 g_0}{M_1 m}$ = constant	$\frac{1}{1.5173 \times 10^7}$	2.818927—10
$\frac{1}{k^2}$ = transformation constant	1.5173×10^7	7.181073
$\frac{M}{M_1}$ = ratio of mass of sun to mass of earth, *Newcomb*	333432.	5.523008
M = mass of the sun	2.0132×10^{33}	33.303878
r = radius of sun for *Auwer's* diameter (31′ 59.26″)	694800 80000 cm.	10.8418603
$r^2 =$		21.6837206
$r^3 =$		32.5255809
p = parallax of the sun, *Newcomb*	8.7965″	0.9443099
D = distance from sun to earth	1493 40870 00000 cm.	13.1741786
r/R_1 = ratio of radii	109.071	2.0377078
S/S_1 = ratio of surfaces $(109.071)^2$	11896.4	4.0754156
V/V_1 = ratio of volumes $(109.071)^3$	1297548.	6.1131234
$G = \frac{\text{gravity at surface of sun}}{\text{gravity at surface of earth}} = \frac{M}{M_1} \cdot \frac{R_1^2}{r^2}$	28.028	1.4475924
ρ_a = mean density of the sun $= \frac{M}{M_1} \cdot \frac{R_1^3}{r^3} \cdot \rho_{a1}$	1.43287	0.156208
v = velocity of the earth in its orbit	18.5212 miles/sec.	1.267670
	29.80670 cm./sec.	6.474314
$f = \frac{v^2}{D}$ = acceleration at the distance of earth	0.59491 cm./sec.	9.774448—10
	0.23422 inch/sec.	9.369615—10
$f = \frac{M}{M_1}\left(\frac{R_1}{D}\right)^2 g_0$ (check)		9.77448—10
$s = \frac{1}{2} f$ = rate at which earth falls toward sun	0.29746 cm./sec.	9.473418—10
	0.11711 inch/sec.	9.068585—10

APPLICATION OF THE THERMODYNAMIC FORMULÆ TO THE GASEOUS ENVELOPE OF THE SUN.

The evidence from the action of the lines in the solar spectrum, as regards shifting, broadening, and reversals, shows that in the envelope resting upon the photosphere, comprising in its contents the reversing layer, the chromosphere, and the inner corona, the gases may be treated as approximately perfect gases and tending to conform to the Boyle-(Mariotte)-Gay-Lussac law, $P v = \frac{K}{m} T$, where P is the pressure in units of force, v the volume, K the absolute gas constant, m the molecular weight, and T the absolute temperature. I propose, also, to apply the same law to the solar mass within the photosphere, with a suitable modification, and to compare the results with the data obtained from the use of Professor Nipher's equations. We can first multiply the equation by any numerical value, x, and distribute the variation between P and T alone, holding the density identical in the two conditions.

Hence,

$$(x P) v = \frac{K}{m} (x T). \tag{77}$$

This asserts that if $\frac{K}{m}$ remains constant, $v = \frac{1}{\rho}$ also remains constant. If a gas, as hydrogen, $\rho = 0.000089996$, is subjected to the same relative increase in P and T, it remains at the same density as that for which its gas constant $\frac{K}{m}$ was computed. We can, therefore, transform hydrogen, or other perfect gases, from terrestrial to solar conditions by simply multiplying by the proper factor. In this case it will be $x = 28.028$, the ratio of g at the surface of the sun to g_0 at the surface of the earth.

In Eclipse Meteorology and Allied Problems, chapter 4, Table 14.—"Fundamental constants," a series of values was computed depending upon assumed values of r, the sun's radius, and G, the ratio between gravity at the surface of of the sun and gravity at the surface of the earth. Since these values have been changed a little in the preceding computations, it will be necessary to reconstruct the numerical values of that table, although the effect upon the dependent quantities is not important. In order that the transition from terrestrial to solar conditions may be made as plain as possible to the reader, we will compute the fundamental constants on the supposition that the earth is surrounded by a hydrogen atmosphere instead of the common air, making allowance for the change in density.

TABLE 8.—*Constants for one atmosphere of hydrogen on the earth.* $p_0 v_0 = R\, T_0 = l$.

Formulæ.	M. K. S. system.		C. G. S. system.	
	Numbers.	*Logarithms.*	*Numbers.*	*Logarithms.*
g_0 = gravity	9.806 m.	0.99149	980.6 cm.	2.99149
ρ_m = density of mercury	13595.8	4.13340	13.5958	1.13340
B_n = merc. col. for 1 atmos	0.760	9.88081	76.0	1.88081
ρ_h = density of hydrogen	0.089996	8.95422	0.000089996	5.95422
$p_0 = \rho_m B_n = \rho_h\, l$ (weight)	10333.	4.01421	1033.3	3.01421
$l = \frac{\rho_m B_n}{\rho_h}$ (hom. atmos)	114815.	5.05999	11481500.	7.05999
T_0 = temperature	273.	2.43616	273.	2.43616
$R_0 = \frac{l}{T_0}$ = gas constant	420.56	2.62383	42056.	4.62383
$v_0 = \frac{1}{\rho_h}$ = specific volume	11.112	1.04578	11112.	4.04578
$p_0 v_0 = l$	114815.	5.05999	11481500.	7.05999
$R_0 T_0 = l$	114815.	5.05999	11481500.	7.05999

TABLE 9.—*Transition to constants for a solar hydrogen atmosphere.* $(G p_0)\, v_0 = R\, (T_0 G)$.

Formulæ.	M. K. S. system.		C. G. S. system.	
	Numbers.	*Logarithms.*	*Numbers.*	*Logarithms.*
$G = g/g_0$	28.028	1.44759	28.028	1.44759
$G p_0 = p$ (weight)	289600.	5.46180	28960.	4.46180
$v_0 = v$ (same density)	11.112	1.04578	111112.	4.04578
$p v = l$	3218000.	6.50758	321800000.	8.50758
$G T_0 = T$	7651.6	3.88375	7651.6	3.88375
$R_0 = R$	420.56	2.62383	42056.	4.62383
$RT = l$	3218000.	6.50758	321800000.	8.50758

TABLE 10.—*Fundamental constants for a hydrogen atmosphere on the sun.*

Data.	Formulæ.	Meter-kilogram-second.		Centimeter-gram-second.	
		Number.	Logarithm.	Number.	Logarithm.
Radius of the sun	r	694800800 m	8.8418603	694800 80000 cm	10.8418603
Gravity acceleration at the surface	$g = G g_0 = 28.028 \times 9.806$	274.843	2.4390843	27484.3	4.4390843
Modulus of common logarithms	M	0.4342945	9.6377843—10	0.4342945	9.6377843—10
Density — Mercury	ρ_m	13595.8	4.1334048	13.5958	1.1334048
Density — Water	ρ_1	1000	3.0000000	1.0000	1.0000000
Density — Air	ρ_0	1.29305	0.1116153	0.00129305	7.1116153—10
Density — Hydrogen	ρ_h	0.089996	8.9542232—10	0.000089996	5.9542232—10
Height of standard barometer	$B_n = \frac{p}{\rho_m} = 0.760 \times 28.028$	21.3013	1.3284060	2130.13	3.3284060
Height of homogeneous atmosphere	$l = \frac{\rho_m B_n}{\rho_h} = R\,T = p\,v_h = \frac{p}{\rho_h}$	3218012	6.5075876	321801200	8.5075876
Barometric constant	$K = \frac{l}{M} = \frac{\rho_m B_n}{\rho_h M} = \frac{R\,T}{M}$	7409746	6.8698033	740974600	8.8698033
Pressure in units of weight	$p = \rho_h\, l = \rho_m\, B_n = \frac{P}{g}$	289608.1	5.4618108	28960.81	4.4618108
Pressure in units of force	$P = \left\{ \begin{array}{l} g\,\rho_h\, l = g\,\rho_m\, B_n = g\,p = P_0\, G^2 = \\ g\,\rho_h\, p\, v_h = g\,\rho_h\, R\, T = \frac{\kappa}{\kappa-1}\frac{l}{T} = \frac{C_p}{A} \end{array} \right\}$	79596670.9	7.9008951	795966709	8.9008951
Press. of one terrestrial atmosphere	P_0	101323.5	5.0057103	1013235 dynes	6.0057103
Volume (specific) of hydrogen	$v_h = \frac{1}{\rho_h}$	11.1116	1.0457768	11111.6	4.0457768
Gas constant for pressure p	$R = \frac{p\,v_h}{T} = \frac{\rho_m B_n}{\rho_h T}$	420.565	2.6238330	42056.5	4.6238330
Gas constant for pressure P	$Rg = \frac{p v_h\, g}{T} = \frac{\rho_m B_n\, g}{\rho_h T}$	1.15589×10^5	5.0629173	1.15589×10^9	9.0629173
Temperature at the photosphere	$T = 28.028 \times 273$	7651.6° C.	3.8837546	7651.6° C.	3.8837546
Temperature gradient	$-\frac{dT}{dh} = \frac{A}{c_p} = \frac{1}{P}$	1.2563×10^{-8}	2.0991049—10	1.2563×10^{-9}	1.0991049—10
Specific heat at constant pressure	$c_p = g\,\rho_h\, A\, R\, T$	186503	5.2706707	18.9968	1.2791478
Heat equivalent of work	$A = \frac{1}{426.8}$ and $\frac{1}{4.1855 \times 10^7}$	0.00234302	7.3697756—10	2.38663×10^{-8}	2.3782527—10
Coefficient from specific heats	$\epsilon_p = \frac{\kappa}{\kappa - 1} = g\,\rho_h\, T$	189261.5	5.2770621	18926.15	4.2770621
Ratio of the specific heats	$\kappa = \frac{c_p}{c_v}$	1.000005	0.0000021	1.000052	0.0000228

The constants are worked out for the meter-kilogram-second (M. K. S.) system and for the centimeter-gram-second (C. G. S.) system, respectively, the formulæ, which are well known, being found in Table 64 of the Report of the Chief of the Weather Bureau, 1898–99, Vol. II.

If hydrogen, as a perfect gas, conforms to the Boyle-Gay Lussac law at so high a temperature as 7651.6, then there must be some stratum in the sun's atmosphere where the density is the same as it is under the standard conditions on the earth. If the gas ceases to be perfect to some extent, this statement must be proportionately modified, but in any case even approximate conditions will be very valuable as giving a general view of the prevailing state of solar physics, in which a footing of some sort is a desideratum for meteorology in general. We next determine the temperature gradient by the computation in Table 10, in which the same constants are employed as above, except that their values have been determined with greater precision.

To obtain the temperature gradient per meter, or the adiabatic rate of fall of temperature per meter, the value of $-\frac{dT}{dh}$ in the M. K. S. system must be multiplied by 1000, and in the C. G. S. system it must be multiplied by 10000 so that they both give

$$(78) \qquad -\frac{dT}{dh} = 0.000012563^\circ \text{ C. per meter, or,}$$

$$(79) \qquad -\frac{dT}{dh} = 0.012563^\circ \text{ C. per 1000 meters.}$$

This can be checked from the terrestrial adiabatic rate, which is 9.86938 per 1000 meters, by multiplying by $\frac{1}{G^2}$.

$$(80) \qquad \text{Thus, } \left(-\frac{dT}{dh}\right)_{\text{sun}} = \left(-\frac{dT}{dh}\right)_{\text{earth}} \times \frac{1}{G^2}.$$

$$(81) \qquad 0.012563 = 9.86938 \times \frac{1}{(28.028)^2}.$$

The rate of the fall in temperature in the atmosphere of the sun is very slow according to this computation, so that variation in the density of the gases is not due so much to changes in temperature as to changes in pressure, which are very rapid, as is shown in Table 11 and fig. 21. The approximate formula

is all that is necessary in this discussion because of the steady state of the temperature just indicated.

Let P_0 = the pressure of 28.028 atmospheres, where h_0, the height, is assumed to be zero.

P = the pressure in atmospheres at the height h.

K = 7409.746 kilometers, the barometric constant.

Then we shall have the reduction formula:

$$(82)\quad \log\frac{P_0}{P} = \frac{h-h_0}{K}, \qquad \text{and} \quad \log P = \log P_0 - \frac{h}{K}.$$

The value of h in seconds of arc is found from

$$(83)\quad 1'' \text{ (second of arc)} = \frac{\text{radius of sun in kilometers}}{\text{radius of sun in seconds of arc}}$$

$$= \frac{694800.800}{16' \times 60 = 960''} = 723.751 \text{ km.}$$

DISTRIBUTION OF THE PRESSURE, TEMPERATURE, AND DENSITY IN A SOLAR HYDROGEN ATMOSPHERE.

Since in a perfect gas $Pv = \frac{P}{\rho} = RT$, we shall have for the density, $\rho = \frac{P}{RT}$. In order to compute R, the gas constant, we take $R = \frac{P}{\rho T}$, where,

P = 28.028 atmospheres,
ρ = 0.089996,
T = 7651.6°, whence we obtain
R = 0.040702 [logarithm = 8.6096146].

The values resulting from the computation are given in Table 11 and fig. 21, "Distribution of the pressure, temperature, and density in a solar hydrogen atmosphere." The indications regarding the prevailing pressure, derived from the behavior of certain lines in the solar spectrum, are that the reversing layer is under a pressure of about 5 atmospheres, or possibly as little as 3 atmospheres (Astrophysics, February, 1896, p. 139; May, 1898, p. 327; April, 1900, p. 240). According to Table 11 the pressure at the height 8″ above the stratum having a pressure of 28.028 atmospheres is 4.636, and this may be adopted as the height of the reversing layer. If the top of the photosphere is 1″ below the reversing layer, the top of the chromosphere 5″ above it, and the top of the inner corona 35″ above the top of the photosphere, then the layer at pressure 28.028 atmospheres is 7″ below the top of the photosphere, and is probably in the midst of the photospheric shell. The temperature gradient is a straight line,[26] but the pressure and density are distributed on curves of the logarithmic type. From 28 to 5 atmospheres the pressure and density change very *rapidly*, but from 2 to 0 atmospheres they change very *slowly*. There is a quick transition in the rate of change

TABLE 11.—*Distribution of the pressure, temperature, and density in the solar hydrogen atmosphere.*

h'' in arc.	h in km.	$\frac{h}{K}$	P	T	ρ	Height of layer $(h-7)''$ above photosphere.	
″						″	
45	32568.75	4.39539	0.001	7242.4	0.000004	38	Top of inner corona.
40	28950.00	3.90702	0.003	7287.9	0.000012	33	
35	25331.25	3.41864	0.011	7333.4	0.000036	28	
30	21712.50	2.93026	0.033	7378.8	0.000110	23	
25	18093.75	2.44189	0.101	7424.3	0.000335	18	
20	14475.00	1.95351	0.312	7469.7	0.001026	13	
18	13027.50	1.75816	0.489	7487.9	0.001605	11	
16	11580.00	1.56281	0.767	7506.1	0.002510	9	
14	10132.50	1.36746	1.203	7524.3	0.003927	7	Top of chromosphere.
12	8685.00	1.17210	1.886	7542.5	0.006143	5	
10	7237.50	0.97676	2.957	7560.7	0.009609	3	
9	6513.75	0.87908	3.703	7569.8	0.012018	2	
8	5790.00	0.78140	4.636	7578.9	0.015031	1	Reversing layer.
7	5066.25	0.68380	5.805	7587.9	0.018796	0	Top of photosphere.
6	4342.50	0.58605	7.270	7597.0	0.023512	—1	
5	3618.75	0.48838	9.104	7606.1	0.029406	—2	
4	2895.00	0.39070	11.400	7615.2	0.036779	—3	
3	2171.25	0.29303	14.275	7624.3	0.045999	—4	
2	1447.50	0.19535	17.875	7633.4	0.057532	—5	
1	723.75	0.09768	22.383	7642.5	0.071955	—6	
0	0	0	28.028	7651.6	0.089998	—7	Within the photosphere.

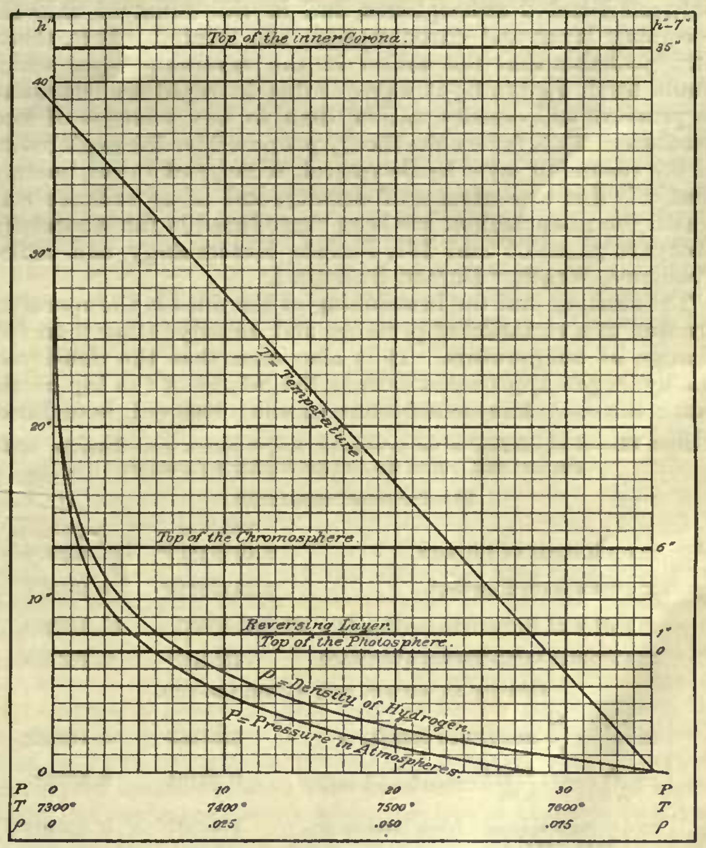


FIG. 21.—Distribution of the pressure, temperature, and density in a solar hydrogen atmosphere.

[26] Since the temperature gradient, $-\frac{dT}{dh} = \frac{1}{P_0}$, was computed for the stratum within the photosphere, where $P_0 = 28.028$, it follows that in the higher strata, in which P has smaller values according to Table 11, the gradient will be greater in the proportion, $\left(-\frac{dT}{dh}\right)\frac{P_0}{P_n}$, assigning successive values to n in the several strata. Hence, the temperature fall is $-\Delta T_n = -\int\left(\frac{dT}{dh}\right)_n dh$, where $\left(\frac{dT}{dh}\right)_n$ increases toward the top of the solar atmosphere. Computations for the successive layers show that the temperature fall is slow up to the level of $P = 1$ atmosphere, beyond which it increases very rapidly as the density diminishes, so that the temperature of space is reached at the top of the inner corona.

We obtained the following temperatures:

Initial stratum in photosphere	$P = 28.028$, $T = 7652°$
Top of the photosphere	$P = 5.805$, $T = 7500°$
Top of the reversing layer	$P = 4.636$, $T = 7450°$
Top of the chromosphere	$P = 1.500$, $T = 6950°$

This law gives too low temperatures at the top of the inner corona to be acceptable at present. Referring to the earth's atmosphere, the law of cooling is not the adiabatic rate, but the gradient is nearly the same as that found for the lower strata in all levels up to 16,000 meters; that is to say, cooling takes place at a uniform rate. The law of cooling in the solar atmosphere is a function which is not now known, and it may fall between the two extreme types indicated above. The entire subject demands a careful research.

between 5 and 2 atmospheres, and in the midst of this the reversing layer and chromosphere are located. It is, therefore, probable that the action in the reversing layer which sends forth visible light waves is due to rapid transmissions in pressure and density, rather than to any changes of temperature. This favors the theory proposed for the explanation of the reversing layer by Becquerel, Wood, and Julius, namely, that it is due to contrasts of density, and in accordance with which the phenomenon has been reproduced in the laboratory. Compare pages 65 and 162, Eclipse Meteorology and Allied Problems, Weather Bureau Bulletin I.

The shifting and the broadening of the lines in the spectrum are due to a variation of pressure and density rather than to a change of temperature. It is also seen that the density of the hydrogen approaches zero at the height of the top of the inner corona. The coincidence in the observed boundaries of these layers in the sun's atmosphere with the results of this computation on the physical state is evidently so perfect as to argue strongly for the correctness of the physical constants employed. The outcome goes to show that the photosphere is the region where great changes in pressure are taking place, so that violent circulations, explosions, and chemical and electrical combinations must prevail, and observations show that this is the case. From the values here employed we can readily compute many other important thermodynamic relations.

TABLE 12.—*Computation of the pressures, temperatures, and densities at the surface and within the sun by Nipher's formulæ.*

		Numbers.	Logarithms.
	Fundamental constants.		
M	= total mass of the sun	2.0132×10^{33}	33.303878
$\frac{1}{k^2}$	= gravitation constant	1.5173×10^{7}	7.181073
r	= radius of the sun in centimeters	69480000000.	10.841860
T	= absolute temperature at surface	7651.6°	3.883755
	Density at surface and within the sun.		
ρ	$=\frac{0.78}{4\pi}\cdot\frac{M}{r^3}=$ surface density	0.37255	9.571182
	$=\frac{\rho_a}{3.86}=\frac{1.43287}{3.86}=$ surface density	0.37121	9.569621
ρ_a	= average density from astronomical data	1.43287	0.156208
r_a	$=0.545\,r=$ distance of stratum ρ_a from center	3.7867×10^{10}	10.578257
v	$=\frac{1}{\rho}=$ specific volume at the surface	2.6842	0.428818
	Pressure at the surface and within the sun.		
P	$=\frac{0.636}{8\pi}\cdot\frac{M^2k^2}{r^4}=$ surface pressure	2.9004×10^{14}	14.462460
P_a	$=5.40\,P=$ average pressure	1.5662×10^{15}	15.194854
r_a	$=0.502\,r=$ distance of stratum P_a from center	3.4879×10^{10}	10.542564
	RT at the surface of the sun.		
RT	$=0.818\,\frac{Mk^2}{2r}$	7.8103×10^{14}	14.892668
	$=Pv$ (The coefficient should be more fully developed)	7.7854×10^{14}	14.891278
R	$=\frac{Pv}{T}=$ the gas constant	1.0175×10^{11}	11.007523
	Temperature at the surface and within the sun.		
T	$=273\times28.028$	7651.6°	3.883755
T_a	$=1.08\,T$	8263.8°	3.917179
r_a	$=0.707\,r$	4.9122×10^{10}	10.691279
	$-\Delta T=8263.8°-7651.6°=612.2°$	612.2°	2.786893
	$+\Delta r=1.000\,r-0.707\,r=0293\,r$ in km.	203577.	5.308728
	$-\frac{\Delta T}{\Delta r}=$ temperature gradient within the sun per 1000 meters	0.0030072	7.478165—10
	Mass of the sun.		
M	$=1.22\,\frac{2RTr}{k^2}=$ mass	2.0091×10^{33}	33.302991
	= Adopted value from Newcomb	2.0132×10^{33}	33.303878
	Weight of 1 gram at the surface of the sun.		
g	$=1.22\,\frac{2RT}{r}=M\frac{k^2}{r^2}$	27428.	4.438178
	$=980.6\times28.028$	27484.	4.439084

It may be observed that the Smithsonian Astrophysical Observatory computes from the Washington observations a tem-

TABLE 13.—*Transformation factor from perfect gases to the material of the sun within the photosphere.*

Formula $P_1=\frac{P_s\rho_h}{\rho_s}$

		Numbers.	Logarithms.
P_s	= surface pressure by Nipher's formula	2.9004×10^{14}	14.462460
ρ_h	= density of hydrogen at surface of sun	0.000089996	5.954223—10
ρ_s	= surface density by Nipher's formula	0.37255	9.571182—10
P_2	= corresponding pressure from inside	7.0065×10^{10}	10.845501
P_1	= pressure found from outside conditions	7.95967×10^{8}	8.900895
F	= transformation factor	88.025	1.944606
R_2	= gas constant for P from Nipher's formula	1.0175×10^{11}	11.007523
R_1	= gas constant for P from hydrogen	1.1559×10^{9}	9.062917
F	= transformation factor	88.025	1.944606

Some such factor as 88 is required to change the conditions outside the photosphere for perfect gases to those inside the photosphere for nonperfect gases or liquids.

TABLE 14.—*Specific heats c_p, c_v, quantity of heat Q, and work W, in the surface stratum of the sun.*

		Numbers.	Logarithms.
ϵ_p	$=\frac{\kappa}{\kappa-1}=\frac{3n-4}{2-2n}$	3.5	
	= assumed value	3.4615	0.539264
A	= heat equivalent of work	$\frac{1}{4.1855\times10^{7}}$	2.378253—10
R	= gas constant	1.0175×10^{11}	11.007523
AR	=	2431.0	3.385776
c_p	$=AR\,\frac{3n-4}{2-2n}=3.5\;AR$	8414.8	3.925040
	Assume 3.4615 *AR*		
$2c_p$		16829.6	
$3AR$		7293.0	
$4AR$		9724.0	
$-\left(\frac{dQ}{dT}\right)_n$	$=+(c_p+4\,AR)$ specific heat due to contraction	18138.8	4.258609
n	$=\frac{2c_p+4AR}{2c_p+3AR}=1.1$ closely	1.1008	0.041699
c	$=\frac{Q}{W}=\frac{c_p+4AR}{2c_p+3AR}$	0.7519	9.876184
	$=0.75$ closely		
W	$=0.636\,\frac{M^2k^2}{2r}=$ work of compression	1.2225×10^{48}	48.087250
Q	$=0.7519\;W=$ heat radiated	0.9192×10^{48}	47.963434
$W-Q$	= excess of work energy over heat energy	0.3033×10^{48}	47.481872
$\frac{Q}{W-Q}$	=	3.03	0.481562
$\frac{W}{W-Q}$	=	4.03	0.605378
c_v	$=c_p-\frac{(8-5n)(4-3n)}{3(2-n)^2(5\kappa-3)}\cdot AR$	7977.2	3.901850
	$=c_p-0.180\;AR$		
κ	$=\frac{c_p}{c_v}=\frac{8414.8}{7977.2}$	1.0548	0.023190

perature of about 6000° for the atmosphere of the sun, although it is quite certain that a higher station, as Mount Whitney, would give a greater temperature, say 6500°. This, of course, takes account of the absorption in the earth's atmosphere, but not of that in the sun's atmosphere. It seems probable that the equivalent of 1000° C. may be absorbed from the stratum included between the midst of the photosphere and the top of the inner corona. If this is not the case, then the outgoing radiation of the sun must be such as to give nearly 4.0 gram-calories per square centimeter per minute on the outer surface of the atmosphere of the earth. The relative absorption in the atmospheres of the sun and the earth, respectively, will be much more readily determined if it can be admitted that the temperature of the sun about 7″ within the photosphere is approximately 7652°. In the following discussion the surface stratum is that which is 7″ below the visible boundary of the photosphere, where the pressure is taken as 28.028 atmospheres. The various comments made by Buckingham and Day as to the value of temperatures extrapolated from terrestrial to solar conditions have their importance, but it is believed that we shall be able to gain a footing by other processes, such as thermodynamic relations, and thereby determine the thermal condition of the sun without such an overstepping of the limits of the actual practicable experiments of the laboratory. We will proceed, in Tables 12 to 14, to consider the conditions within the solar mass, with the aid of Nipher's formulæ, and to show that here, too, there is ground for encouragement, because of the numerous agreements between two independent sets of data, namely, the astronomical quantities and the thermodynamic values.

DISCUSSION OF THE VALUES DERIVED FROM TABLES 12 TO 14.

Table 12, "Computation of the pressures, temperatures, and densities at the surface and within the sun by Nipher's formulæ," contains a series of values at the surface stratum in the photosphere, where the pressure has been taken at 28.028 atmospheres as the result of external conditions. These have now been computed from astronomical data M, k^2, r, and the assumed temperature 7651.6. The purpose is to compare these two sets of values, one computed from external conditions, and the other from the internal conditions, the former for strictly perfect gases, as hydrogen, and the latter from such non-perfect gases or liquid material as makes up the body of the sun. While the law $Pv = RT$ applies to perfect gases, we may yet obtain some approximate idea of the state of the sun inside the photosphere if a transformation factor can be found by which to pass from the first system to the second. In a circulating mass like the sun it is probable that something like this law applies throughout the mass. At any rate the view can be tested to some extent by studying the two sets of data. There is, of course, some danger of arguing in a circle through so complex a system of formulæ, but I think that the general conditions herein exhibited conform more closely to a natural solar mass than the results heretofore derived by the use of Ritter's formulæ.

The density.

The average density of the sun from astronomical data is 1.43287, and it is a denser liquid than water. The surface density is 0.37255, or about one-fourth the average density. This latter occurs at the distance $0.545r$ from the center of the sun, and if anything like the same gradient of density is maintained throughout, the density near the center of the sun is not far from 5.7, which is about the mean density of the earth. We may, therefore, assign a more or less solid nucleus to the sun, which becomes viscous at a distance of about one-third the radius from the center, and soon thereafter mobile. The transitions within the sun are gradual, but at the photosphere there is apparently a mixture of liquid and gaseous masses in active transitions, and these seem to be the conditions indicated by the phenomena observed in the sun spots. The prominences, faculæ, and chromosphere are strictly in a gaseous atmosphere; the photosphere is a mixture of gases and liquids, and the interior consists of a circulating liquid passing into a solid nucleus near the center. While the sun's pressure by gravitation alone would increase the density of its constituents, the temperature is at the same time high enough to balance this tendency to compression, so that the material in the sun is in about the same state as the material of the earth, except that here the outer layers have advanced toward solidification under the prevailing low temperature. A contracting sun, in order to keep up its radiation, must be circulating freely, and this precludes a very high degree of viscosity, except near the center.

The pressure.

Beginning with a pressure of 28.028 atmospheres in that layer of the photosphere where the temperature is 7652°, which on the sun is equivalent to 7.96×10^8 dynes, we compute that for a hydrogen gaseous envelope the pressure practically vanishes at the top of the inner corona. Beyond this layer, into which hydrogen is ejected in the prominences, the conditions are favorable for all the electrical and magnetic phenomena belonging to the cathode rays in rarefied gases. At the photosphere, where the materials change from gases to vapors and liquids, there is a corresponding equivalent increase in pressure up to 2.90×10^{14} dynes. It would take this increase in pressure to pass from the gaseous to the fluid state at the high temperature there prevailing. If a fluid may be considered as a gas brought by pressure at a given temperature to the liquid condition, then this pressure difference also represents the explosive energy when the liquid changes to a gas. If the liquid is elevated from the interior to the surface of the sun by convection currents, then, on reaching the surface, it may greatly expand and even explode when vaporization takes place, as is commonly observed on the edge of the sun through the enormous velocities measured by the change in wave lengths, by the Doeppler principle, or by anomalous dispersion. Within the body of the sun, at the distance 0.5 radius from the center, the pressure is 1.57×10^{15} dynes, which is 5.4 times as much as at the surface. By the same ratio, the pressure would be eleven times as much at the center, though this law doubtless changes within the nucleus. The pressure is comparatively uniform below the sun's surface, and widely discontinuous at the surface. Hence, the convectional currents and the dependent phenomenon of rotation in latitude are leisurely motions compared with the explosive action at the surface layers.

The temperature and the gas constant.

Nipher's coefficients are carried to only three decimals, which is doubtless sufficiently accurate for the determination of the value of the contractional constant n. It is not quite sufficiently accurate, however, to give proper check values from one formula to another, but I have not thought it worth while to carry this computation beyond the approximate stage. If we pass from a perfect gas to a fluid, the value of the gas constant adopted must be interpreted as merely suggesting important relations, and too much emphasis must not be laid upon certain obvious criticisms which naturally arise. We may suppose that the mass of the sun beneath the photosphere, while apparently fluid or viscous, yet moves in accordance with the general law, by reason of convection, so that it is continually readjusting itself to conform somewhat closely to this general law of gaseous elasticity. At any rate, this is the theory upon which we have proceeded in the discussion. We compute the product RT by Nipher's formula, and check it with the product $\frac{P}{\rho}$ found from the pressure and density, and then with the temperature $T = 7651.6°$ find $R = 1.0175 \times 10^{11}$ for the fluid of density 0.37255 in the surface layer. The temperature within the sun

at the distance 0.707 radius from the surface becomes 8264°, and at this rate, an increase of 612° in 0.293 radius, the total increase from the surface to the center is 2089°, making the central temperature 9741°. This gives an average gradient of —0.0030072° per 1000 meters from the center to the surface. We find, also, the gradient from the photosphere to the top of the inner corona to be —0.012563° per 1000 meters. The gradient of the temperature is about four times as great in the atmosphere of the sun as inside the photosphere. The cooling is, therefore, more rapid outside than it is inside the photosphere.

The mass of sun, the weight of 1 gram on the surface of the sun, and the transformation factor.

The mass of the sun is 2.0091×10^{33} by Nipher's formula, agreeing closely with that adopted from Newcomb, 2.0132×10^{33}, the former being computed through the product RT, and thus checking all the quantities. The weight of 1 gram at the surface of the sun is 27428 by Nipher's formula, through the product RT, and this agrees with the simple product $g = 980.6 \times 28.028 = 27484$, thus checking again. The transition factor from a perfect gaseous system to that actually existing at the surface, where the density is 0.37255, is found as indicated. We find the pressure corresponding to 0.37255 instead of that for which the computation was made in a hydrogen atmosphere of density 0.000089996, and obtain $P_2 = 7.0065 \times 10^{10}$ through Nipher's formula, as if the atmosphere were of the greater density. For the actual hydrogen atmosphere we computed (Table 13) $P_1 = 7.95967 \times 10^{8}$. Hence, $P_2 = 88.025\ P_1$, so that 88.025 is the required factor. Similarly, the gas constant from Nipher's formula is $R_2 = 1.0175 \times 10^{11}$. It was computed for the actual hydrogen atmosphere (Table 13) to be $R_1 = 1.1559 \times 10^{9}$. Again, $R_2 = 88.025\ R_1$, so that there is mutual agreement. Some such factor as 88 is required to pass from the law for perfect gases, $P_1 v = R_1 T$, to that for solar liquids, $P_2 v = R_2 T$.

It will not be advantageous to speculate as to what this factor 88 signifies, but it is not so large as to be improbable in passing from a gaseous to a fluid state, as it may stand for the internal forces of viscosity or friction and molecular cohesion, and possibly for some unknown forces of electricity and magnetism.

Specific heats, energy of radiation, and contraction.

Carrying the values of the several quantities through the various formulæ we find that they conform to the prescribed conditions, as follows:

Specific heat of contraction —	$\left(\frac{dQ}{dT}\right)_n$	=	18138.8
Exponent and coefficient	n	=	1.1 closely.
Heat energy of radiation	Q	=	0.9192×10^{48}
Work energy of contraction	W	=	1.2225×10^{48}
Ratio $\frac{\text{heat radiated}}{\text{work of gravitation}}$	$= c = \frac{Q}{W}$	=	0.75 closely.
Ratio $\frac{\text{heat radiated}}{\text{excess}}$	$= \frac{Q}{W-Q}$	=	3.00 closely.
Ratio $\frac{\text{work of compression}}{\text{excess}}$	$= \frac{W}{W-Q}$	=	4.00 closely.
Specific heat at constant pressure	c_p	=	8414.8
Specific heat at constant volume	c_v	=	7977.2
$\kappa = \frac{c_p}{c_v}$ ratio of the specific heats at the temperature 7652°		=	1.0548

We note that this ratio $\kappa = \frac{c_p}{c_v} = 1.4065$ in terrestrial conditions; in solar conditions inside the photosphere $\kappa = 1.0548$; and in the hydrogen envelope $\kappa = 1.000052$ according to the preceding discussion.

Surveying this set of interrelated thermodynamic values, and especially in view of the fact that they seem to conform so well with the known astrophysical conditions derived from observation, and with the astronomical data obtained by the general laws of motion, we conclude that they afford ground for further research. If they form the approximate basis for a sound solar physics they will become important in further meteorological studies.

Chart XII A. Average monthly vectors of the general circulation in the West Indies at the various cloud levels. First arrangement.

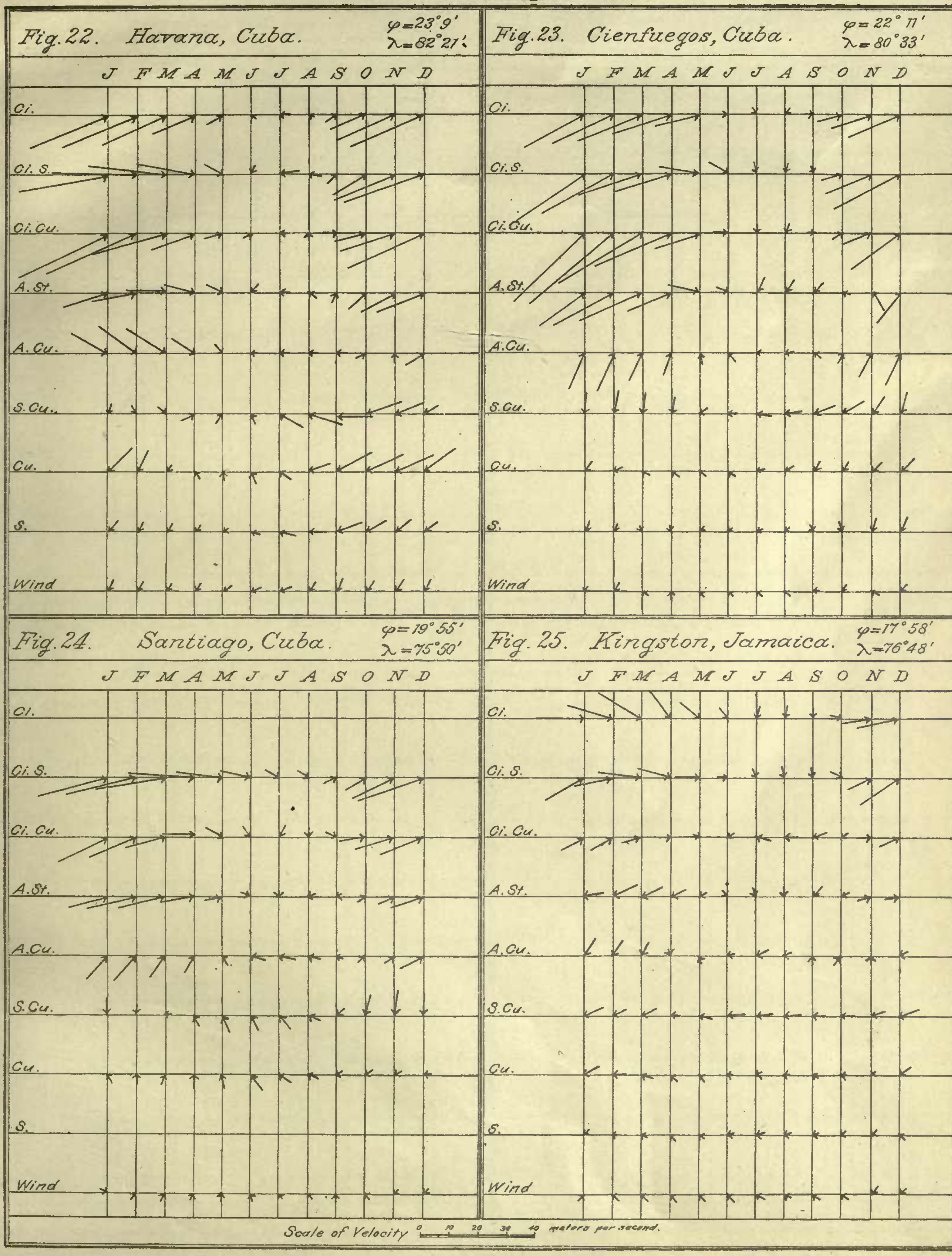


Chart XII B. Average monthly vectors of the general circulation in the West Indies at the various cloud levels. First arrangement.

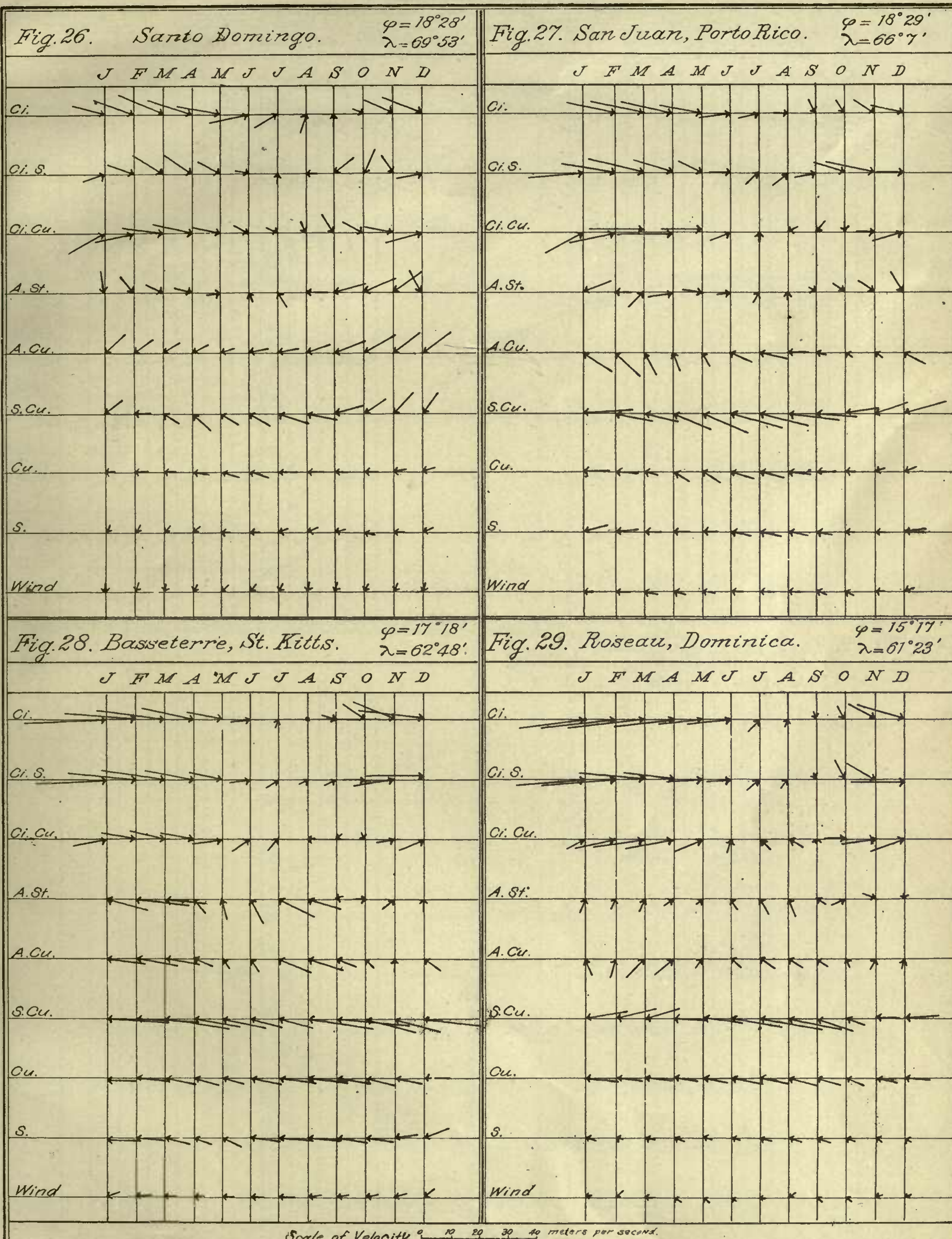


Chart XII C. Average monthly vectors of the general circulation in the West Indies at the various cloud levels. First arrangement.

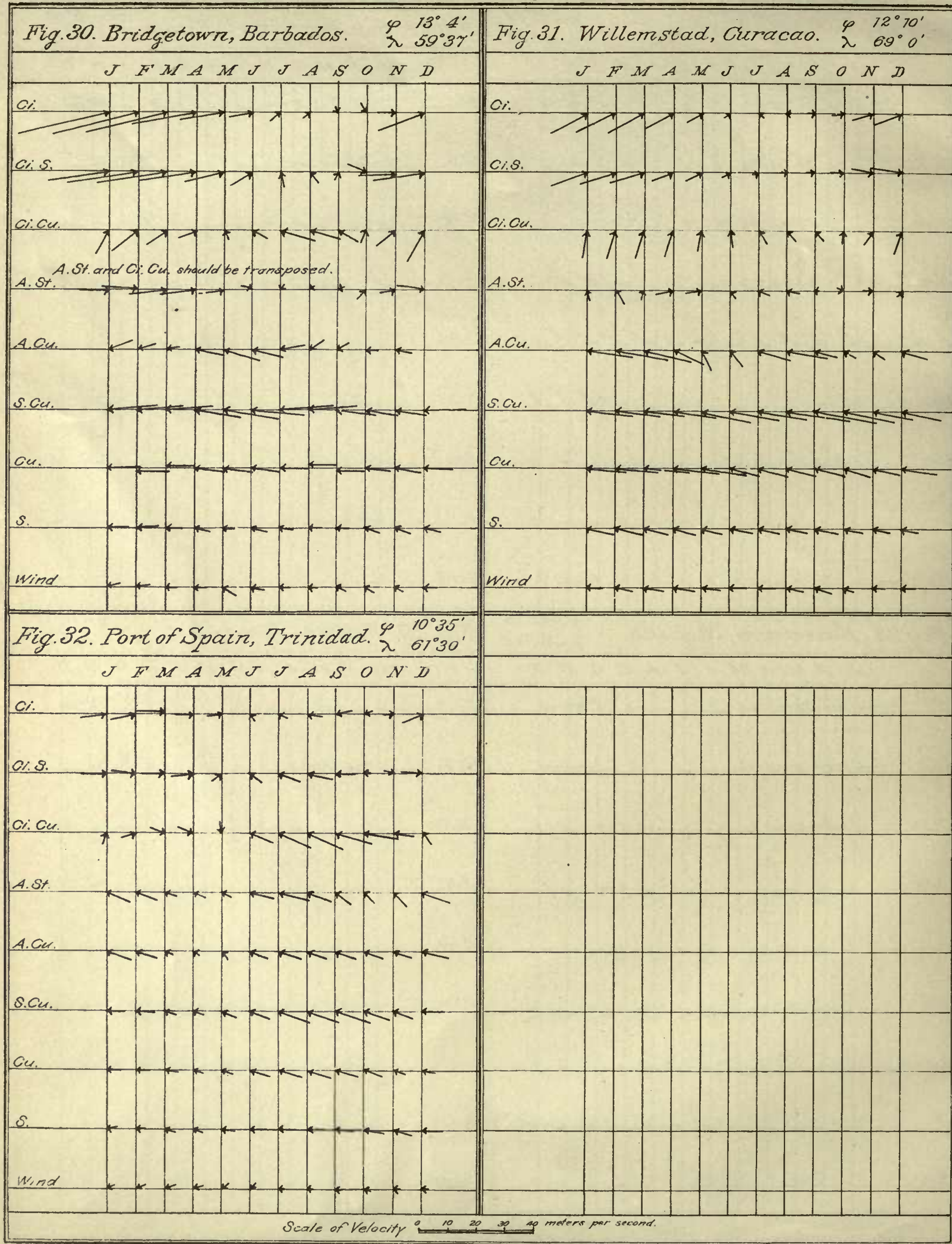


Chart XIII A. Average monthly vectors of the general circulation in the West Indies at the various cloud levels. Second arrangement.

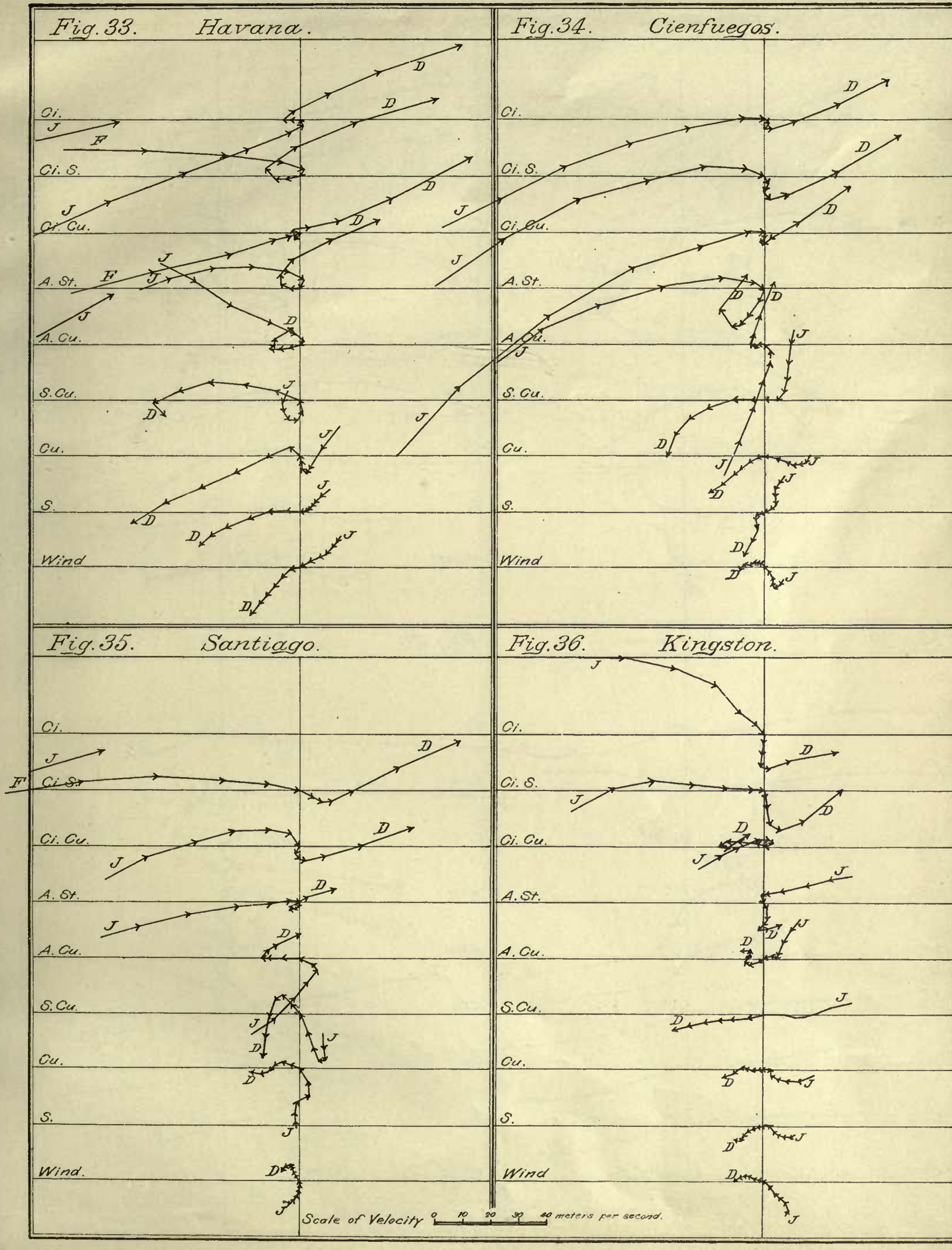


Chart XIII B. Average monthly vectors of the general circulation in the West Indies at the various cloud levels. Second arrangement.

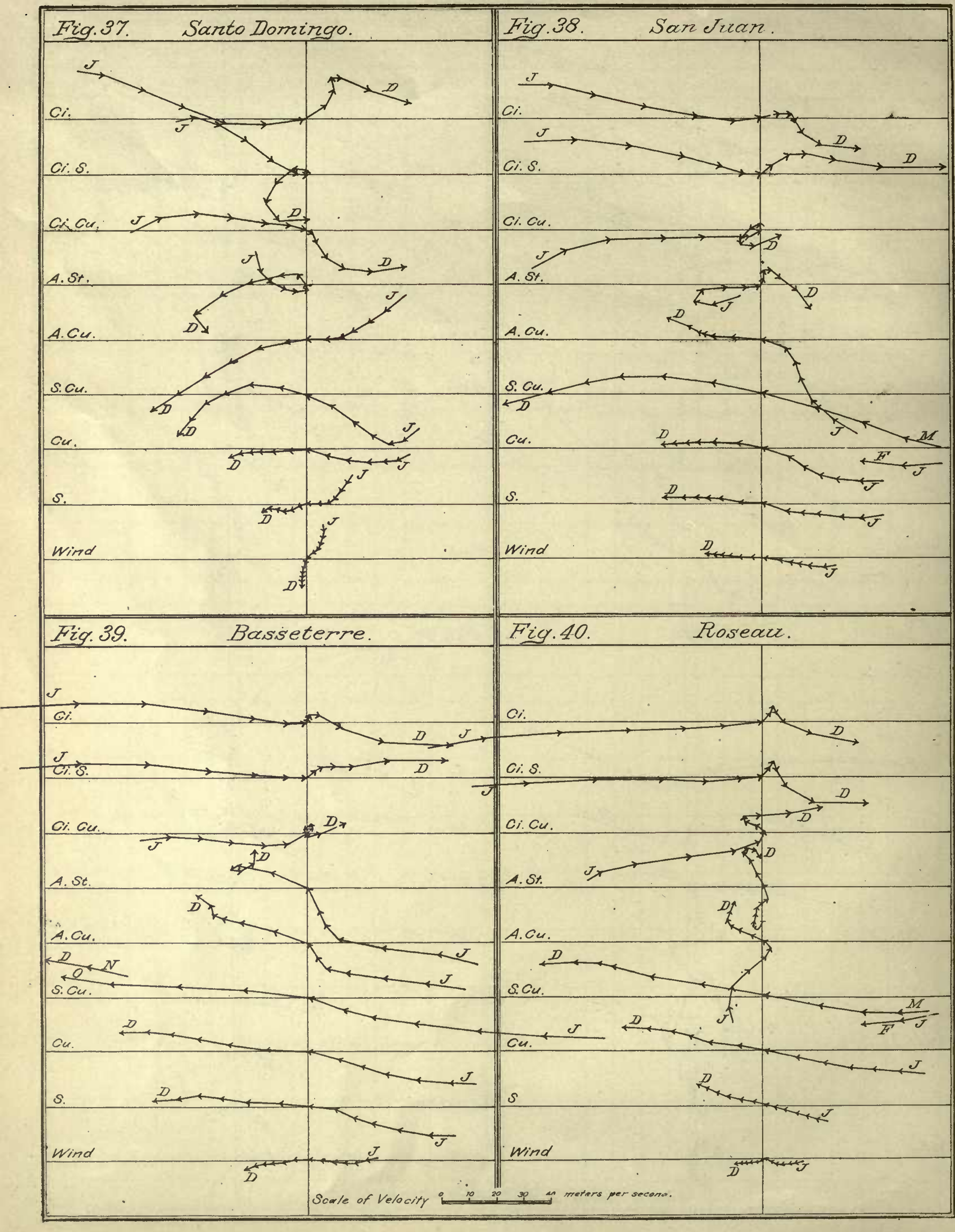


Chart XIII C. Average monthly vectors of the general circulation in the West Indies at the various cloud levels. Second arrangement.

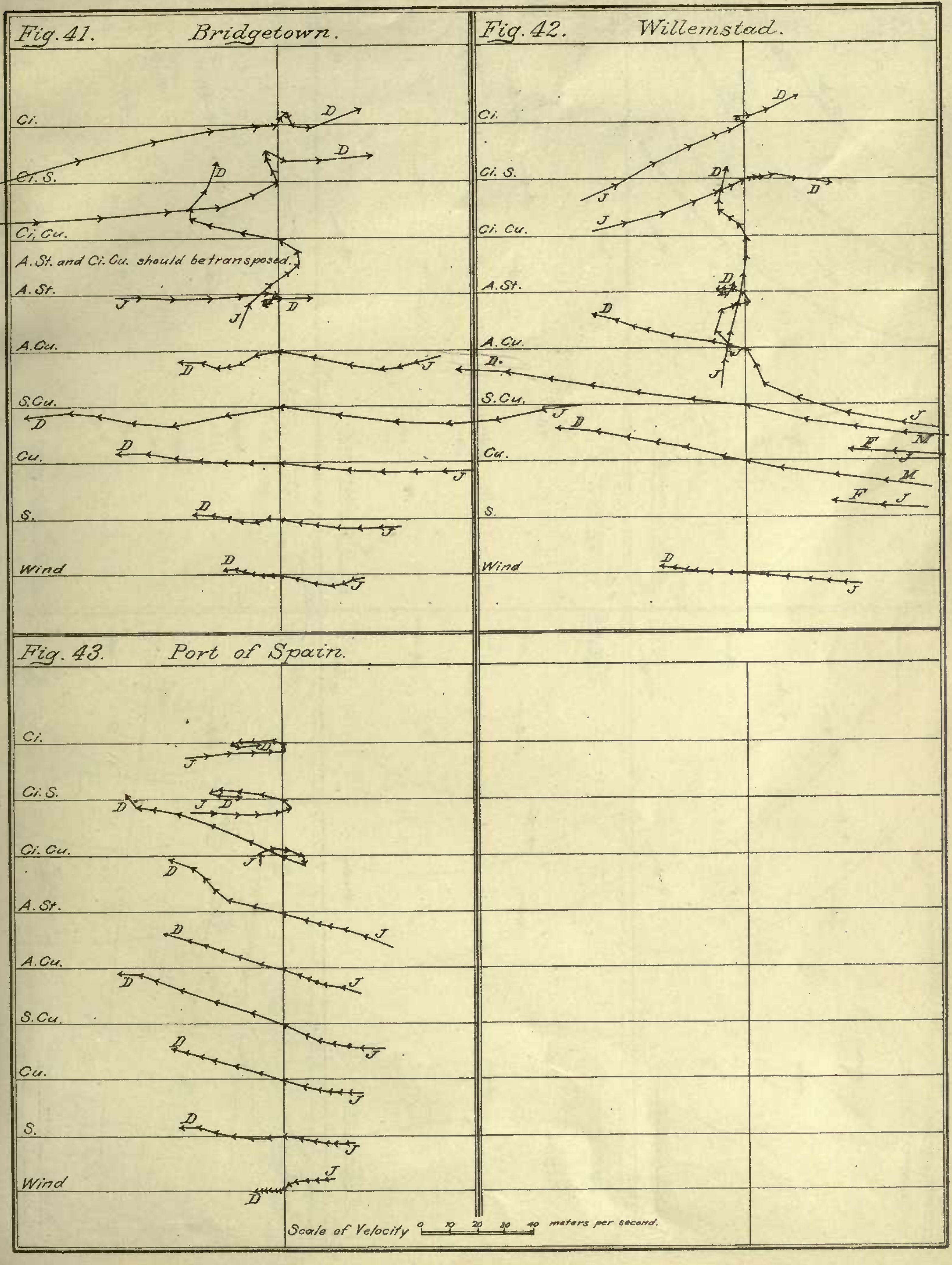


Chart XIV A. Average seasonal vectors of the general circulation in the West Indies at the various cloud levels.

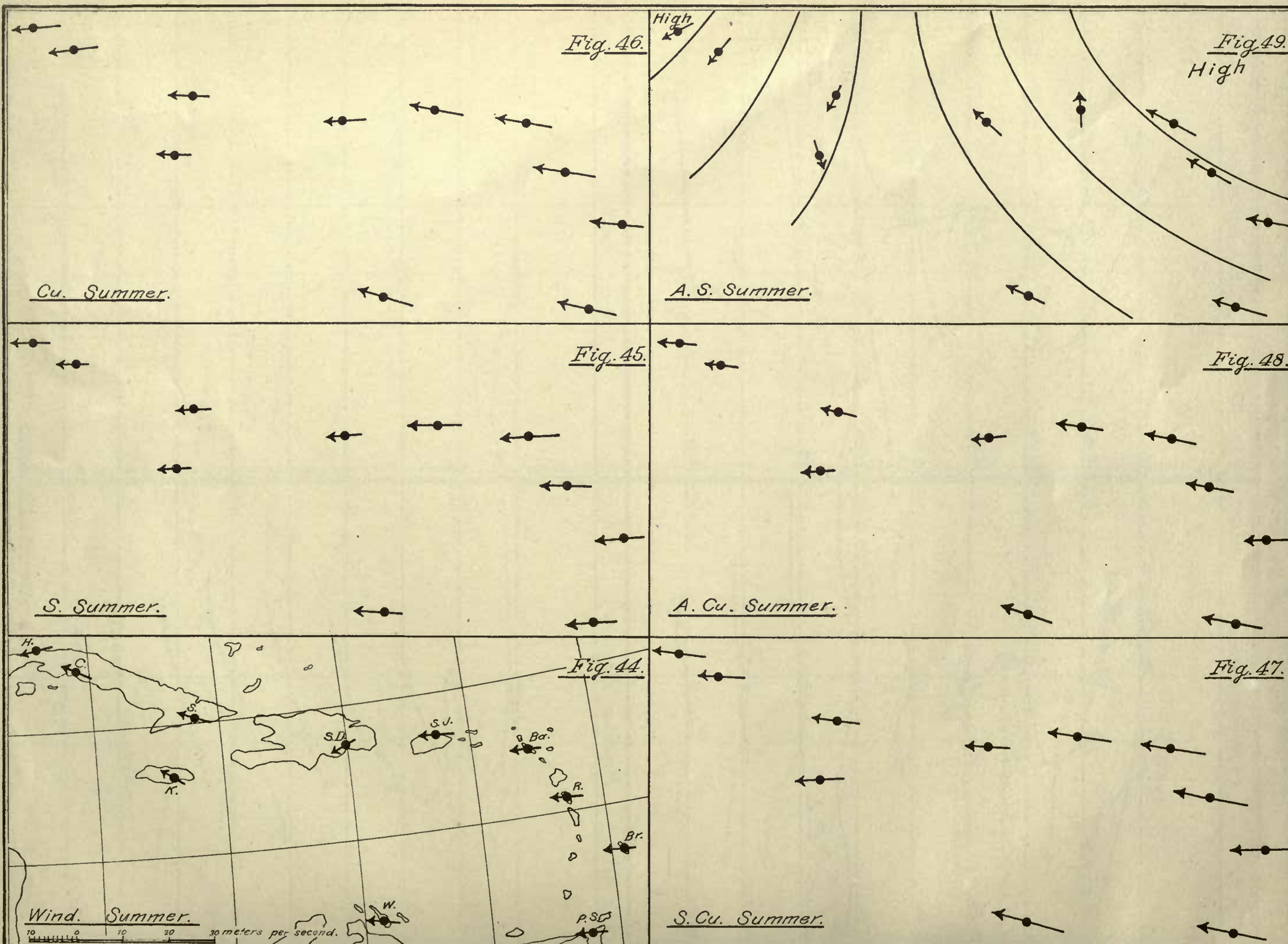
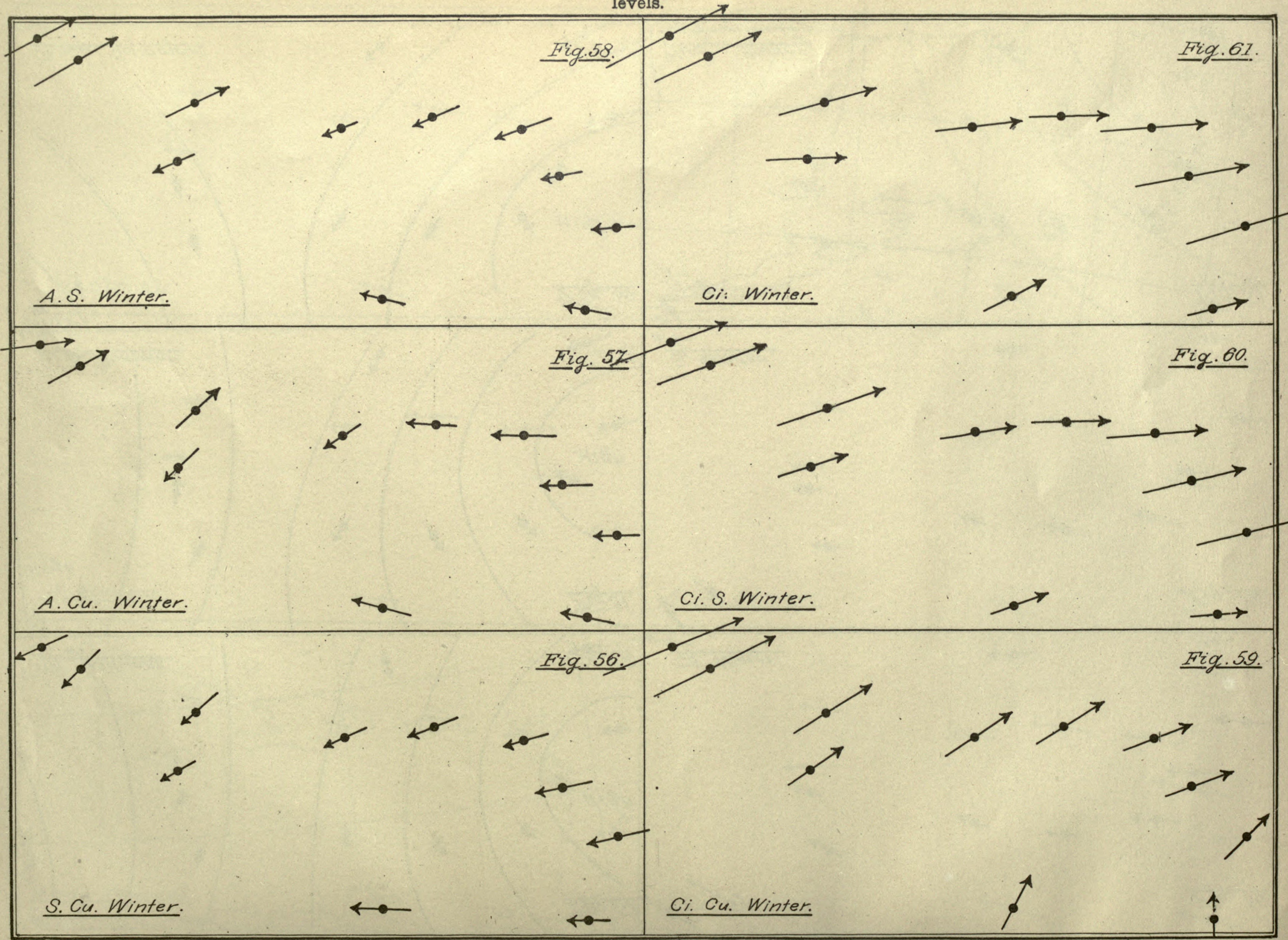


Chart XIV B. Average seasonal vectors of the general circulation in the West Indies at the various cloud levels.

Chart XIV C. Average seasonal vectors of the general circulation in the West Indies at the various cloud levels.



V.—RESULTS OF THE NEPHOSCOPE OBSERVATIONS IN THE WEST INDIES DURING THE YEARS 1899–1903.

METHODS OF OBSERVATION AND REDUCTION.

The observers of the United States Weather Bureau occupied eleven stations in the West Indies during the years 1899–1903, and the opportunity was utilized to make a survey of the motions of the atmosphere in that region of the Tropics by means of nephoscopes.

The instruments were of the Marvin pattern, and the method of observation, to obtain the azimuth and velocity of motion, was identical with that described in the Report of the Chief of the Weather Bureau, 1898–99, Vol. II, chapter 2. The reductions were, however, carried out more perfectly than in any previous research of the kind in the following manner: (1) The three readings at each observation were reduced to a mean azimuth and velocity, for entry on the computing sheets. (2) Each vector V, φ was separated into its rectangular components, $+ S, + E$, where S is positive southward on the meridian, and E is positive eastward on the parallel of latitude. (3) The algebraic sum of each set of components was then taken from month to month, and the groups for corresponding months during the years 1899–1903 combined. (4) These sums were divided by the number of observations, to obtain the mean velocity component per observation. The observations were taken for two years in the afternoon hours, and for the other two years in the forenoon hours, so that the diurnal variation was practically eliminated by putting the entire data for each of the twelve months in a single summation. (5) These mean ordinates were plotted month by month for the nine cloud levels, and average curves were drawn through them. In spite of the fact that the number of observations is large, when taken as a whole, yet, when subdivided among so many strata, there is more scattering of the ordinate points in certain levels than is desirable. The average curves approximate the normal components which would be derived from a very much more extensive series of observations. The full report will contain the observed ordinates and those obtained by graphical adjustment. (6) The resultant polar coordinates V, φ were then computed from the rectangular coordinates. Up to this point the numerical data had been carried forward in the number of millimeters passed over in twenty-five seconds on the scale of the nephoscope. (7) These numbers were now reduced to velocities in meters per second, by multiplying them by the factor $\frac{1}{5}$ H, where H is the adopted height of the cloud stratum. The values of H were adopted from the Washington cloud heights, after considering the heights obtained at Manila during the same cloud year, 1896–97. These velocities and azimuths were transferred to charts drawn to the scale 1 millimeter = 1 meter per second. In preparing these charts for publication in the Monthly Weather Review, the original scale has been reduced to 0.6 millimeter = 1 meter per second, as shown by the "velocity scale in meters per second" at the bottom of each chart of the sets Chart XII and XIII, the scale of Chart XIV being unchanged.

CHARTS OF THE RESULTING VELOCITIES AND DIRECTIONS OF MOTION FOR THE WEST INDIES.

The vectors V, φ have been plotted for each station, so that the mutual relations of the resulting motions can be properly compared and studied. There are several remarks to be made about the observations themselves. At Willemstad the observers have frequently misnamed the *cumulus* clouds as *cumulo-stratus.* This becomes apparent on plotting the vectors. The angles are correct, but the length of the arrows is too great. I have accordingly interpreted the values of V_1 and V under strato-cumulus as belonging to the cumulus level, and have used the reduction factor 0.5 instead of 0.9 in drawing the charts.

At Bridgetown the vector systems of the alto-stratus and the cirro-cumulus levels have apparently been interchanged. As they now stand at Bridgetown they are inconsistent with the flow of air as determined at Basseterre, Roseau, Port of Spain, and Willemstad; but if they are transposed, then there is harmony. The observation sheets indicate that the observers have an unusually large number of cirro-cumulus entries and comparatively few alto-stratus, so that apparently they were accustomed to name many alto-stratus clouds as cirro-cumulus clouds. It is not easy to secure identical estimates of cloud forms at so many independent stations as we have used, and these few instances of apparent discrepancies are gratifying evidence of the general excellence of the results in other respects.

On comparing the charts here presented with those published by Prof. H. H. Hildebrandsson for the international committee, it appears that he has computed only the angle of the azimuth of motion without the velocity, and that the same schematic velocity is entered throughout the year. Apparently the actual velocities have not been computed for that report at any station. The importance of having the velocity as well as the direction of motion is evident, and this is emphasized by examining the great variations between the summer and the winter velocities and between those at the different cloud levels of each station of the West Indies. Comparing the Hildebrandsson and the Bigelow results for Havana, Plate VII, International Report, with fig. 22, Chart XII A, the azimuth directions are not in agreement in the autumn; comparing those for the Antilles, Plate III, International Report, with figs. 28, 29, 30, Charts XII B, and XII C, Basseterre, Roseau, and Bridgetown, the legend "Nuages superieurs" should apparently be changed to "Nuages intermediaries" or "Nuages inferieurs."

The vectors of the West Indian stations have been plotted in three forms: The first, Charts XII A, XII B, XII C, figs. 22 to 32, wherein the vectors of the same month throughout the nine levels terminate on the same vertical line; the second, Charts XIII A, XIII B, XIII C, figs, 33 to 43, wherein the vectors for June terminate on the same vertical line and the others in succession, so as to form a continuous broken line; the third, Charts XIV A, XIV B, XIV C, figs. 44 to 61, showing approximate normal vectors for winter and summer. The first enables us to study the movements simultaneously occurring in a given month from the surface wind to the cirrus level, and from this many important conclusions can be drawn. The second makes more distinct the general course of the movement in the several strata throughout the year, and especially the nature of the currents that depend upon the forces producing the westward drift of the lower levels and the eastward drift of the upper levels, together with the transition levels between them. The third system of charts is a composite of the mean winter and the mean summer systems, respectively, some of the minor irregularities being rectified in the adopted vectors. January and February constitute the middle of the winter group, while July and August are at the middle of the summer group. In adopting these vectors regard was had to the most probable balanced system which is indicated by the entire set of vectors. If the reader has doubts as to the accuracy of these final results, the original material of the third set of charts is to be found in the first and second sets, or in tables from which all the charts have been plotted, which will appear in the full report. Numerous studies in the dynamic meteorology of the Tropics are now practicable for the first time, but as it will require much careful labor to execute them, only some general remarks are required in this place.

THE ARCH SPANNING THE TROPICS, WHICH DIVIDES THE EASTWARD DRIFT FROM THE WESTWARD DRIFT OF THE GENERAL CIRCULATION.

The general theory of the circulation of the atmosphere shows that in the temperate zones of the Northern and the Southern hemispheres there is a strong prevailing eastward component producing an eastward drift, while in the Tropics there is a prevailing westward component causing a westward drift. The tropical westward drift is, however, limited in altitude, and at a certain elevation the drift reverses from a westward to an eastward direction. The position of the curve which separates the eastward from the westward drift varies with the season of the year. When the sun is far to the south and the northern winter prevails, the arched curve must be skewed to the south, and when the sun is far to the north and the northern summer prevails the arch is skewed toward the north. This is on the assumption that the foot of the arch rests on nearly the same latitude in winter and summer at any given region of the earth. The high pressure belt, which fixes the position of the arch on the surface of the earth, for the eastern portion of the United States lies somewhere between +30° and +35° north latitude, and crosses the Atlantic coast at about Florida and South Carolina. It is desirable to determine from our observations its exact location, but as this is of subordinate importance for our immediate purpose it can be passed over in this connection. Further accounts of the mathematical significance of the tropical discontinuous surface between the prevailing eastward and westward drifts may be found by consulting the full report, and my paper in the Monthly Weather Review, January, 1904, Vol. XXXII, p. 15.

One special purpose of the West Indian nephoscope survey for 1899–1903 was to determine this surface of separation in the higher levels, and its variation with the season of the year. The velocities and direction of motion on either side of it, and the numerous meteorological inferences that can be drawn from these conditions made the work of primary importance in the development of the dynamics of the atmosphere. The series of Charts, XII A to XIV C, figs. 22 to 61, are now available for this purpose. The twelve months, as observed, may be taken in two groups, of which the winter group is distributed about January and February as the central months, and the summer group about July and August. If there had been simultaneous observations in the Southern Hemisphere at latitudes corresponding to those that were occupied by the West Indian stations in the Northern Hemisphere, we should then have the data applying simultaneously to the entire tropic zone. The same result can be closely approximated by treating the winter group as representing the north tropical zone, and the summer group as representing the south tropical zone, during the northern winter. Hence, by using the results from November to April in the northern latitudes, and assuming that during this period the conditions observed for the northern latitudes during May to October then prevailed in the southern latitudes, the synchronous action of the circulation can be found for the northern winter throughout the Tropics. By reversing this process the corresponding results for the northern summer are obtained.

An inspection of the vectors of motion in the winter months shows that for Havana, Cienfuegos, and Santiago, Cuba, with mean latitude 22°, the reversal is about midway between the cumulo-stratus and alto-cumulus levels, or at approximately 3500 meters above the sea level. At Kingston, Santo Domingo, San Juan, Basseterre, Roseau, and Bridgetown, having the mean latitude of 17°, the reversal is apparently in or above the alto-stratus level, or about 6400 meters elevation. At Willemstad and Port of Spain, with mean latitude of 12°, the reversal is in the cirro-cumulus level, at about 8000 meters elevation. This is shown on the northern winter branch of fig. 62, "Mean altitudes at which the westward drift reverses to the eastward drift in the Tropics."

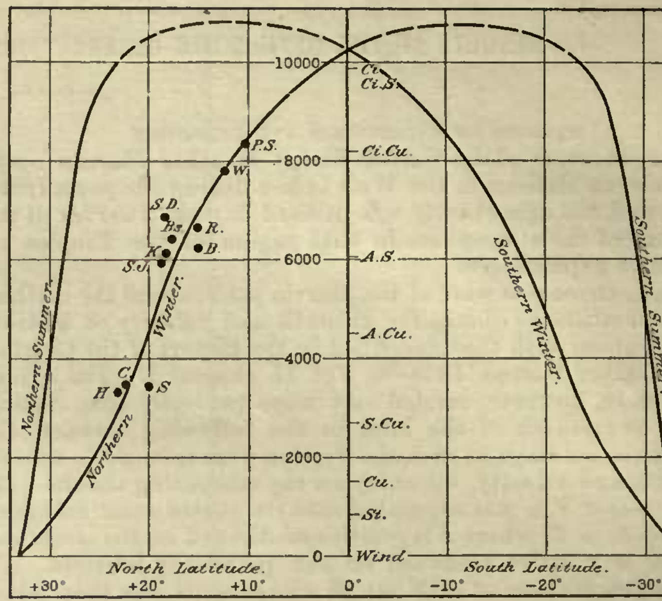


Fig. 62.—Mean altitudes at which the westward drift reverses to the eastward drift in the Tropics.

An examination of the vectors for the summer group of months indicates that generally there is a tendency to reversal in the cirrus and cirro-stratus levels, at about 10,000 meters elevation. This, however, is not definitive, and I have indicated the probable top of the arch in a broken line rather than to positively assign the limit. The summer westward circulation is very feeble throughout the column from the sea level at least to the 10,000-meter level, and it may be that it extends even to a higher level over the effective thermal equator, which lags about forty-five days behind the position of the sun in latitude. In the very complex circulation actually existing in the Tropics it is not possible to make very exact statements regarding such a loosely defined boundary as that which really separates the eastward and westward currents of the atmosphere underneath the sun. The calms of the thermal equator move northward and southward with the sun, and apparently the westward drift extends upward unbroken to about six miles, though at that level there are evidences that the eastward drift is making itself felt. The east and west currents play against each other at these high levels in a somewhat irregular manner.

THE LEVELS OF MAXIMUM HORIZONTAL VELOCITY.

An examination of Charts XIII A, XIII B, XIII C, figs. 33 to 43, brings out clearly the levels of the maximum horizontal currents. The circulation generally increases in westward velocity from the surface to the strato-cumulus level, then falls off to a minimum in the alto-stratus level, where the direction is irregular, and then increases to a maximum eastward velocity in the cirrus level. The stations of Cienfuegos and Santiago, Cuba, together with Kingston and Santo Domingo, give relatively small resultant velocities in the lower levels, from wind to cumulus, as compared with Havana, San Juan, and the southeastern group, Basseterre, Roseau, Bridgetown, Port of Spain, and Willemstad. The former group of stations develop greater irregularity in their azimuth directions than do the latter group, and consequently their resultants are much diminished in magnitude. This probably indicates some action of the continental mass of North America in disturbing the westward drift, which prevails steadily in the Windward Islands, and in changing its general direction from the southeast, which is the natural flow from the trades, to the northeast in that region. The strato-cumulus level in the more eastern stations has a powerful westward current, which falls off decidedly in the vicinity of Cuba. On the other hand the north-

LIBRARY OF THE UNIVERSITY OF CALIFORNIA

eastward or eastward velocity in the three upper levels—cirro-cumulus, cirro-stratus, and cirrus—is at a maximum over the Cuban stations, and tends to diminish toward the southeast; that is, from the Antilles to Port of Spain. The trade from the southeast holds quite uniformly in the lower levels of the eastern group of stations, and the northeastward upper trade prevails in the upper levels of the western group. Between the lower and the upper levels there is a region of transition whose nature is quite clearly indicated. At Port of Spain the southeast trade prevails throughout the year with maximum in the cumulo-stratus level, diminishing and partially reversing in the cirro-stratus and cirrus levels. At Willemstad, Bridgetown, and Roseau there is an exclusively northern component in the alto-cumulus and alto-stratus levels. At Basseterre and San Juan this component becomes northwestward or shows signs of reversing. At Kingston and Santiago the azimuths are irregular and the velocities small, and at Cienfugos and Havana the eastward drift practically dominates. Beyond these statements it is not very safe to go at present. The circulation is complex and depends largely upon the relation of the Atlantic high area, belonging to the general high pressure belt of the Northern Hemisphere, to the adjacent continents. The normal system seems to be like that of Willemstad, Bridgetown as corrected, and Roseau, where the movements from the southeast in the lower levels change to movements from the south in the middle levels and from the southwest in the higher levels. The southwest antitrades are conspicuous over Cuba, but they become west and even northwest antitrades over San Juan and the Windward Islands. One is surprised to find that the southeast trades prevail so steadily in the northern zone, winter and summer, even up to latitude $+17°$, including Port of Spain, Willemstad, Bridgetown, Roseau, Basseterre, and San Juan. It is evidently of primary importance that meteorologists should extend this nephoscope survey to the Azores, Ascension Island, St. Helena, the South American Continent, Central America, and Mexico. The entire circulation of the atmosphere can thus be carefully determined by means of the method here illustrated.

THE WINTER AND THE SUMMER CIRCULATIONS.

At a glance the great contrast between the motions of the air in winter and summer is apparent. Over Havana the northeastward velocities are above 30 meters per second in winter, and in the summer they become about five meters per second and are directed westward. The summer vectors on Chart XIII A, XIII B, and XIII C form an irregular broken line which in some cases become a very good loop. This looping or tangle in the line is quite characteristic of the summer vectors in the middle levels at Havana, Cienfuegos, Santiago, Kingston, San Juan, Basseterre, and of the vectors in the high levels at Roseau, Bridgetown, Port of Spain, and Willemstad. These loops and tangles sometimes make it difficult to determine from the original observations what is the true mean curve to be drawn, because the ordinates are quite scattering and irregular in the middle cloud levels. In the lower and higher levels there was little difficulty experienced from this cause in drawing the mean lines. As a general principle none of the broad statements which meteorologists have been accustomed to make regarding the trade-wind system seems to hold over a very large region. The changes from one locality to another are numerous and important, showing that the circulation of the Tropics is really very much localized. There exists no system of cyclones and anticyclones to disturb the general circulation, but this circulation is itself much more complicated than it is in the temperate zone. The dynamics of the two systems are very different, and depend upon a complex distribution of temperatures and pressures. Every effort should be made to determine what these are before resorting to analytic discussions, which will be of little permanent value until all the principal facts are known.

THE CAUSE OF THE WEST INDIAN HURRICANES.

The hurricanes which devastate the southeastern districts of the United States in the months from July to October originate in the West Indian region, and, as much conjectural writing has been published in order to account for them, it is important to throw what light is possible upon the subject. In the complex circulation shown to exist over the Carribean Sea, it is easy to suppose that gyratory local circulations can be set up which will develop into cyclonic action. The summer circulation is irregular, as befits a belt of calms such as prevail in the doldrums, or it has a feeble westward direction. In the winter this motion has become powerfully eastward in the upper levels, in consequence of the overspreading of the cold sheet of air from the temperate zone, which is controlled by the eastward drift of higher latitudes. In the autumn, especially in September, a marked change takes place, by which the stagnant or westward moving air is sharply propelled eastward. This is seen by examining the months of August, September, and October, in the alto-stratus and the higher levels on Charts XIII A, XIII B, XIII C, figs. 33, 34, 35, 38, 39, 40, and 41, for Havana, Cienfuegos, Santiago, San Juan, Basseterre, Roseau, and Bridgetown. This indicates the locality where hurricanes are especially generated, and agrees with otherwise well known facts. They seldom occur as far south as Port of Spain and Willemstad, but at these stations there is no indication of a sharp reversal in the cloud region. *The levels from alto-stratus to cirrus, from four to six miles high, are those chiefly concerned in causing the hurricane formation.* The lower levels do not have the same reversal currents, but their vectors are very steadily directed from the southeast to the northwest throughout this season of the year. A hurricane is built up on exactly the same mechanical principle as a tornado, namely, by the conflict of two currents flowing together from different directions and having different temperatures, only the hurricane is much deeper than the tornado, the hurricane forming a tube from four to six miles long, while the tornado tube seldom exceeds one mile in length. In the tornadoes of the United States the cool wind from the northwest flows against and over the warm current from the south or southeast as they meet in the central valleys. Between them, at the height of about one mile, a vortex tube is formed, which, by its gyratory action, extends downward through the lower strata, which latter must be in a more or less quiescent state or else drifting slowly forward from top to bottom. In the case of the hurricane in the high levels we have the cool eastward drift of autumn strengthening and spreading into the tropic zone, with a northeastward or eastward current, as shown at Havana, Cienfuegos, Santiago, San Juan, Basseterre, Roseau, and Bridgetown, Charts XIII A, XIII B, and XIII C. This meets the southeast trade with currents moving northwestward, as shown at Willemstad and Port of Spain, Charts XIII C, figs. 42 and 43, and between them a gyration is set up which penetrates downward four to six miles, and so produces a vortex tube of large dimensions and great power at the surface, such as hurricanes exhibit. This conclusion is an exact agreement with the results obtained in the Report of the Chief of the Weather Bureau, 1898–99, vol. 2, as given on chart 35 for the tropical hurricane and described on page 457. It was there shown that the vectors of motion in the cirrus level require the existence of a vortex tube at least five miles long. The usual drift of hurricanes from the place of generation is at first westward or northwestward, and this is because they are carried along with the prevailing currents in the lower and middle levels. It seems then that hurricanes build up in the higher levels by the counterflow of currents there prevailing, that they penetrate through four to six miles of lower strata to the surface, and are borne along westward by entanglement in the lower currents through which they penetrate. When these change their direction to the northward and northeast-

ward the hurricane track recurves with them. On the other hand the hurricane itself disappears in higher latitudes and is transformed into a shallow cyclone, because there the countercurrent flow in the higher levels ceases. These conclusions can be further illustrated by reference to Charts XIV A, XIV B, and XIV C, mentioned above.

APPROXIMATE NORMAL CIRCULATION IN THE WEST INDIES DURING THE WINTER AND SUMMER, RESPECTIVELY.

On Charts XIV A, XIV B, XIV C, figs. 44 to 61, which show the average normal circulation in the West Indian district of the Tropics, special attention is directed to the vectors in the four upper levels—alto-stratus, cirro-cumulus, cirro-stratus, and cirrus—for the summer months, figs. 44 to 52. These charts were drawn by inspecting all the available data from the eleven stations and carefully determining the most probable mean vectors that would make a natural, well-balanced system, wherein irregularities due to imperfect observations would be rectified. A comparison with the vectors of Charts XII A, XII B, and XII C shows that the changes which have been introduced are all of a minor nature, and it is supposed that a larger number of observations with the nephoscopes would produce a system of vectors very closely approximating those here adopted. In the lower levels, from the surface wind up to and including the alto-cumulus level, the currents are similar, except that in the strato-cumulus level the velocity is at a maximum. From this level it diminishes both upward and downward.

It should be remembered that in discussing the nephoscope observations of 1896–97 for the strictly cyclonic and anticyclonic components in the circulation of the middle latitudes, we reached the same result regarding the prevailing level of maximum velocity, namely, that the maximum velocity is in the strato-cumulus level. Compare chart 68, page 625, Report of the Chief of the Weather Bureau, 1898–99, Vol. II.

In the upper levels of the Tropics, on the other hand, a new circulation is prevailing, which is peculiarly interesting in connection with the causes that generate hurricanes. Instead of one single westward drift, as in the five lower levels, there exist two countercurrents in the four upper levels. The western group of stations—Havana, Cienfuegos, Santiago, and Kingston—have their vectors pointing southward; the eastern group of stations—that is, Santo Domingo, San Juan, Basseterre, Roseau, Bridgetown, Port of Spain, and Willemstad—have vectors pointing generally northward. Between them there is a distinct region of counterflow, and, consequently, an area of low pressure. If we assume that in the upper strata, where the mechanical friction is a very small quantity, and where the internal mixing from local minor cyclones is negligible, the vectors are directed nearly parallel to the isobars, then we can easily construct their configuration, though we can not assign numerical values to them without further investigations. On the eastern side there is a high area, which is a portion of the western end of the prevailing Atlantic high pressure. On the western side there is another high pressure area, whose origin is not so easy to understand. Over the North American Continent in summer the heated surface conditions produce a general low pressure area in the lower strata, and simultaneously a high pressure area in the upper strata. It is very likely that the western high pressure in the upper strata over the West Indies is really the southern extension of the continental high pressure area prevailing in summer over the United States. Some further computations on our nephoscope observations in the United States will be required to determine the exact facts.

Between these two high pressure areas in the West Indies there exists a low pressure area, with countercurrents on either side, so that all the conditions are present that are needed to produce a *cyclone in the upper strata.* If the prevailing pressures and currents become intensified at any time, the high-level cyclone is strengthened, and it then penetrates with its large vortex tube to the surface as a regular hurricane. The entire circulating structure is borne along northwestward in the prevailing drift of the *lower* levels till it recurves in the southeastern part of the United States. It is evident that the locality of the formation of the center of cyclonic motion may shift eastward and westward over the West Indian region, depending upon the state of the atmosphere at the time, the position of the two great high pressure areas, and the conflicting currents in action. The normal type here produced is in reality made up of numerous fluctuations on either side of the mean. In forecasting for hurricane conditions it becomes necessary to watch carefully the motions of the four upper cloud levels, in order to learn the practical signs foreshadowing such a hurricane condition.

On Charts XIV B, XIV C, figs. 53 to 61, "Normal vectors for winter," the interest is of a different character from that explained in connection with the summer type. Here it is the reversal from the westward drift of the lower strata to the eastward drift of the upper strata. From the surface up to and including the strato-cumulus level the configuration is generally the same throughout the West Indian region, Then the reversal vectors first set in at the western stations, Havana, Cienfuegos, Santiago, in the alto-cumulus and alto-stratus levels; the other stations become involved later in the higher cirro-cumulus, cirro-stratus, and cirrus levels, where the regular antitrades prevail. The azimuths of the higher vectors show that the northward component nearly vanishes in the cirrus level over the eastern stations. It will be necessary for meteorologists to outline the eastern portions of the Atlantic high area in the levels up to six miles before executing conclusive discussions of the important dynamic problems suggested by these vectors.

VI.—THE CIRCULATION IN CYCLONES AND ANTICYCLONES, WITH PRECEPTS FOR FORECASTING BY AUXILIARY CHARTS ON THE 3500-FOOT AND THE 10,000-FOOT PLANES.

In my paper on "The mechanism of countercurrents of different temperatures in cyclones and anticyclones," MONTHLY WEATHER REVIEW, February, 1903, some account was given of the construction of the auxiliary charts of barometric pressures for the United States on the 3500-foot and the 10,000-foot planes, to correspond with the daily weather map on the sea-level plane. These new charts have been prepared daily since December 1, 1902, and they have been carefully studied from that time with two purposes in view, the results of the examination being briefly stated in this paper, while the more detailed explanation will appear in Volume II of the Annual Report of the Chief of the Weather Bureau for 1903–4. The first purpose concerns the information they have given as to the actual circulation in the strata above the surface, and its relation to several theories which have been advanced to account for these local circulations, and the second has regard to the derivation of precepts useful in forecasting the weather. It is quite impossible, I presume, to convey to one who has not had an opportunity to see these upper-level charts any adequate impression of their significance to modern meteorology, or of the transformations which take place in the structure of the three systems of isobars, as a cyclone passes over the United States. They must be taken together for the best results, and the study of their *mutual* configurations and variations affords us an insight into the true cause of storm formation, which is decisive as to their nature, and is of especial interest to the intelligent forecaster.

THE STRUCTURE OF THE ISOBARS AT DIFFERENT LEVELS.

In the MONTHLY WEATHER REVIEW for January and February, 1903, several examples were given of the configuration of the isobars in cyclones on the three reference planes, and also of their resolution into two components, namely, the normal isobars of the month and the abnormal or disturbance isobars, which, when added to the normal isobars, produce the observed isobars of the date. The normal monthly isobars were taken from the Barometry Report, 1900–1901, and the separation of the two systems was made by means of a graphical construction. Our purpose was to separate the strictly local disturbance circulation from the general circulation, so far as the isobars are concerned, and to compare this component of the pressure with the wind vectors which had been derived from the cloud observations of 1896–97, a summary of which was given in the MONTHLY WEATHER REVIEW for March, 1902. To illustrate this process, Charts XII and XIII for February 3, 1903, are introduced.

Chart XII, fig. 63, gives the sea-level isobars as on the weather map for February 3, 1903; fig. 64 gives in black the isobars of the same date on the 3500-foot plane, and in red those on the 10,000-foot plane. The components are given on Chart XIII, where the black lines on fig. 65 give the normal system for February on the 3500-foot plane undisturbed by cyclonic action, and the red lines the abnormal system, which, when added to the normal, produces the black lines of fig. 64 The black lines in fig. 66 give the normal and the red lines the abnormal system on the 10,000-foot plane. Since the disturbance on the sea-level plane is not much affected by the normal system, the resolution into components is omitted. It follows that we shall properly compare together fig. 63 and the abnormal systems, or red lines, on figs. 65 and 66 when discussing the theory of cyclonic gyrations. In the course of the year numerous modifications of the fundamental type occur, but in all cases it is not difficult to detect what this modification is and to be certain that we are dealing with one simple, natural structure to which every theory must conform to become acceptable.

In order that we may concentrate attention more closely upon the primary structure, which suffers numerous modifications in the local circulation, an example is given on Chart XIV of a typical system of the normal and of the abnormal isobars, such as occur in a well developed cyclone for the month of February, upon which to base certain conclusions that are in fact sustained by the entire series, without any sort of contradictory or conflicting evidence. Chart XIV, figs. 67, 68, and 69 give the normal, and figs. 70, 71, and 72 the abnormal isobars on these planes.

The discussion can not be considered complete without joining with the isobars the corresponding systems of isotherms at all three levels, but in the present stage of our study it is not possible to do this with accuracy in the higher levels. The temperature conditions in the upper strata can not be reached by direct computation, as has been done with the isobars, until a very much more extended series of actual measurements than we now possess has been made by means of balloon and kite ascensions, such as are proposed at the Mount Weather Observatory. On this account the resolution of the isotherms into their normal and abnormal exponents is now limited to the sea-level plane, or rather, to the surface of the United States. An example is taken from the map of February 27, 1903, which is reproduced on Chart XV, figs. 73 and 74, where the isotherms are printed in red. The temperature components are formed by exactly the same method as was employed in resolving the isobars, the normal temperature system being taken from the Barometry Report. Fig. 73 gives the weather map, and fig. 74 the normal and abnormal isotherms.

THE GEOMETRICAL CONSTRUCTION OF HIGH AND LOW PRESSURE AREAS.

From the study of the isobars on the three planes, it is possible to draw several important conclusions which have the value of general principles. It was shown in the MONTHLY WEATHER REVIEW for February, 1903, fig. 25, "The formation of local anticyclones and cyclones in the general circulation about the poles," that the distribution of pressure commonly observed can be approximately reproduced by the superposition of systems of concentric circles, representing positive and negative local additions to the general circles of the hemis-

phere concentric about the pole. The charts of the year 1903 give the actual shape of the lines as they occur in nature, which are seldom concentric circles in their form. In the cyclone they more nearly resemble ellipses, which as a family have *one* focus in common, the other retreating as the area of the ellipse enlarges. In other words the isobars are crowded together on one side of the cyclone, as the northeastern, and opened on the opposite side. I explain this fact from the circumstance that the warm area and greater bouyancy of the air is on the crowded quadrant, so that a stronger tendency to true vortex action exists there than on the cold side, where the downward flow of the air tends to diminish vortex motion, that is to decrease the abnormal pressure gradients. This can be verified by examining the abnormal isotherms of Chart XV, for February 27, 1903. The major axis of the system of ellipses is pointed forward of the axis of symmetry of the entire circulation, generally the meridian, and makes an angle α with it. All possible relations of the ellipses to the axes chosen, as $x =$ the meridian, positive southward, $y =$ the parallel, positive eastward are contained in the equation,

$$(1-e^2 \sin^2 \alpha)\, y^2 - 2\, e^2 \sin \alpha \cos \alpha\, xy + (1 - e^2 \cos^2 \alpha)\, x^2 + (2\, e^2\, d \sin \alpha - 2\, b)\, y + (2\, e^2\, d \cos \alpha - 2\, a)\, x + (a^2 + b^2 - e^2\, d^2) = 0.$$

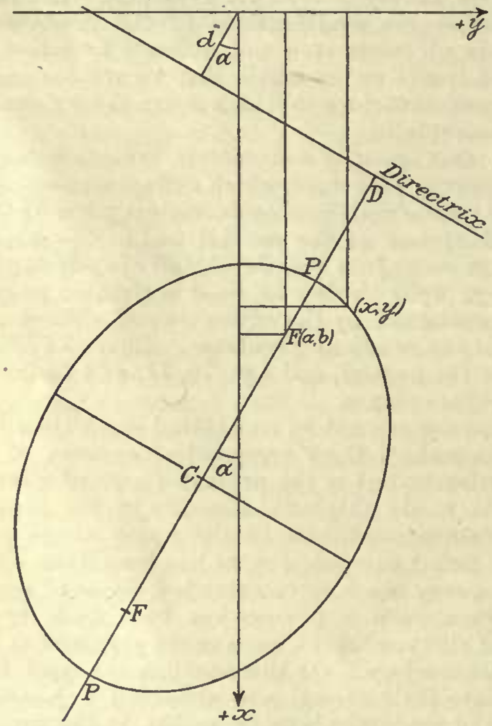


FIG. 75.—The general ellipse.

Let $(a\ b) =$ the coordinates of the focus F.

$(x\ y) =$ the coordinates of any point on the curve.

$d =$ distance of the directrix.

$\alpha =$ the angle that transverse axis $(PP = A)$ makes with x.

$A =$ the length of the transverse axis.

$B =$ the length of the conjugate axis.

$e =$ eccentricity $= \left(\dfrac{A^2 - B^2}{A^2}\right)^{\frac{1}{2}}$.

$FC = Ae =$ distance focus to center.

$FP = A\,(1 \mp e) =$ distance focus to vertex.

$PD = \dfrac{A}{e}(1 \mp e) =$ distance vertex to directrix.

$CD = \dfrac{A}{e} =$ distance center to directrix.

A much simplified case occurs where $d = 0$, $\alpha = 0$, where the directrix is the axis y, and the transverse axis coincides with the axis x. For example, if $e = 0.57$, $d = 0$, $a = 10$, $b = 0$, $\sin \alpha = 0$, $\cos \alpha = 1$, the equation becomes,

$$y^2 + 0.67\, x^2 - 20\, x + 100 = 0.$$

The solution gives such point pairs as (6.4, 0), (8, 4.14), (10, 5.75), (12, 6.60), (15, 7.01), etc., from which the ellipse is to be plotted.

To illustrate the composition of two systems of isobars we take that of right lines and circles.

Let $R =$ the radius of the circle,

$(a, b) =$ the coordinates of the center,

$(x, y) =$ the coordinates of any point on the circle.

The general equation of the circle is,

$$(x-a)^2 + (y-b)^2 = R^2.$$

If we take $b = 0$ and transpose the terms,

$$y^2 = -x^2 + 2\, a\, x + R^2 - a^2.$$

The equation of the condition of the isobar which is the resultant of successive circular abnormal isobars added to successive right line normal isobars, is that the sum of certain pair numbers shall be constant on the same line. Thus, $A + B =$ constant, where $A = n\,x =$ some multiple, n, of the coordinate x, and $B =$ the gradient number on the circles. For example, take the gradient on the normal right lines one-half that on the normal circles, so that $n = \frac{1}{2}$, which is about the average in highly developed storms. Take successive circles, $R = 6, 5, 4, 3, 2$, whose gradient numbers are respectively $B = 0, -1, -2, -3, -4$. Take $a = 6$, $A = \frac{1}{2}x$, $A + B = 0$ for the 0-line and $n = \frac{1}{2}$.

TABLE 15.—*Form for computing the coordinates of the resultant curve.*

R.	B.	$A = \frac{1}{2}x$.	$y^2 = -x^2 + 2\,a\,x + R^2 - a^2$.	y coordinate.
$R = 6$	$B = 0$	$x = 0$	$y^2 = 0 + 0 + 36 - 36 = 0$	$y = 0$
$R = 5$	$B = -1$	$x = 2$	$y^2 = -4 + 24 + 25 - 36 = 9$	$y = \pm 3.00$
$B = 4$	$B = -2$	$x = 4$	$y^2 = -16 + 48 + 16 - 36 = 12$	$y = \pm 3.47$
$R = 3$	$B = -3$	$x = 6$	$y^2 = -36 + 72 + 9 - 36 = 9$	$y = \pm 3.00$
$R = 2$	$B = -4$	$x = 8$	$y^2 = -64 + 96 + 4 - 36 = 0$	$y = 0$

Similarly, by taking the proper groups of R, B, x, for the $-1, +1, -2, +2, \ldots\ldots$ lines in low and high areas, we obtain the coordinates of the resultants. The completed computation is shown on fig. 76. "Right lines and circles, where the gradients are twice as great on the circles as on the right lines." The preceding example plots the 0-line of the low area as will be seen by the pair points (0, 0), (2, ± 3.00), (4, ± 3.47), (6, ± 3.00), (8, 0).

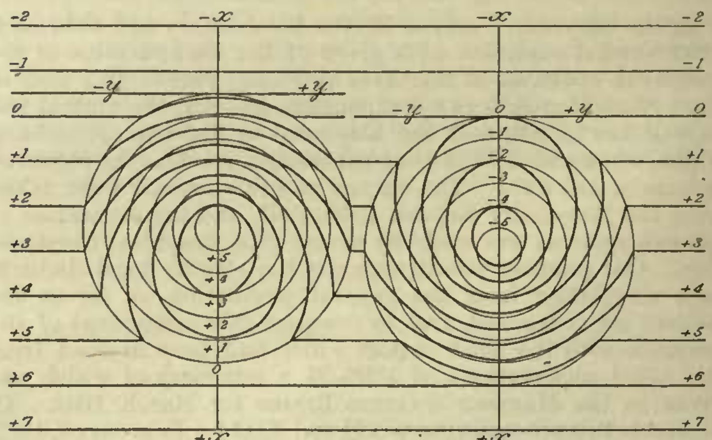


FIG. 76.—Right lines and circles where the gradients are twice as great on the right lines.

In this manner the simple cases can be readily handled analytically, and the principal is theoretically to be extended to all such groupes of curves as can be reduced to a mathematical expression. It is, however, evident that we can not obtain the equations for the observed isobars except in simplified cases, and that generally a graphical solution is all that can

be employed. Practically, one takes a sheet of normal isobars and places over it a sheet of observed isobars. Then, the differences at the points of intersection are marked in the sense that so many tenths of an inch must be added to or subtracted from the normal isobar to produce the observed isobar at that point. By joining up the points of equal difference numbers the system of the abnormal isobars for the day is obtained. The axes of the negative areas, *L*, have one nearly equal angle, *a*, with the meridian, but the axes of the positive areas, *H*, *H*, become convergent upon two cusps, *C*, *C*, fig. 70, Chart XIV, which tend to unite over a saddle, *S*, of relatively high pressure, separating the cyclone proper *L* from the wide spread region of low pressure lying beyond the axis.

THE CUSP FORMATION AND ITS CHANGES.

Between the isobars marked 0 and —1, in all levels, there is a line of pressure which is exactly the same throughout its extent, as indicated by the line of dots on fig. 11, 3500-foot level, MONTHLY WEATHER REVIEW, January, 1903. The rounded cusps of the typical abnormal isobars of Chart XIV become sharp cusps at that pressure, in contact at a central point on the saddle, and from this line the pressure falls in two directions, but rises in two other directions as shown on the typical figures. It is evident that by raising or lowering the pressure of the entire cyclonic region, the number of the closed curves inside the cusps can be diminished or increased. For instance in intensifying the cyclone the existing cusps advance and flow together, and then separate into an additional closed curve and an additional line at the top of the figure. If the pressure is diminishing, an inclosed curve advances to meet an outside line, and joining with it produces a new cusp, but at the sacrifice of an inside closed isobar. Thus, there is continued building or destroying of the closed central isobars going on in the action of the cyclones and anticyclones of the atmosphere in proportion to the energy of the circulation at any given level. Now, on passing from one level to another along the same vertical we find a similar increase and decrease of the cusp action, showing that in the same cyclone this difference of strictly cyclonic circulation exists.

The general rule is that *the number of closed isobars steadily diminishes with the height,* as shown on Chart XIV. Our maps give this structure in all stages of the development, from energetic storms with power to penetrate to considerable heights, to shallow storms which have become entirely depleted within two miles of the ground. In the winter the cyclonic circulation is exclusively in the lower strata, and is soon stripped away by penetrating the swiftly moving general circulation of the eastward drift. A remarkable fact has been developed, namely, that as the warm weather comes on and the power of the eastward general currents diminishes, the structure of the cusps and closed isobars is maintained at very much higher elevations. Thus, in April and May the 10,000-foot level is as much involved as the 3500-foot level is in January and February. I explain this by two facts: first, that the general currents in the lower levels of January and February have retreated to higher elevations in April and May, carrying the cyclonic structure with them; second, that in the warm months there is much more surface heat to dispose of in cyclonic action than in the winter, but that it must seek higher levels to find the cold air necessary to bring about the thermal equilibrium.

In the case of hurricanes, as shown in Paper No. V of this series, the cyclonic structure is powerful at the height of the cirrus levels, five to six miles above the surface, and this is in confirmation of the results of the Weather Bureau cloud observations of 1896–97, chart 35. It was shown in the same report that, taking the entire year, the maximum cyclonic circulation is in the strato-cumulus level, two miles above the surface, whereas in the winter it is lower and in the summer higher than that level. The cusp structure then diminishes with the height, but there is *no instance in which there is any sign that the closed isobars of low pressure reverse into closed isobars of high pressure over the same center.* The closed isobars are of the same sign till they are depleted and wiped out by penetration into the eastward drift. This is a conclusion without contradiction, and it is fundamental to cyclonic theories. Since the cyclonic circulation has an inward component, as is well known to be the fact, in the lower levels, it follows that it must have an *inward component in all levels* until it is absorbed in the upper strata. There is no reversal of the gradient system of isobars in the higher level as compared with the lower level, and *there can be no outflow in the upper level of the cyclone proper* unless it can be shown that there is a reversal of the isobars. The theoretical discussions which assume a reversal of gradients in the upper portions of the cyclones have no foundation in these observations, and all such observations as claim to have found in the cloud vectors of the upper levels a true outflow (Blue Hill, Hildebrandsson and others) have apparently not made the separation between the general and the cyclonic vectors with sufficient precision to escape this incorrect inference. It should be remembered that the Weather Bureau has reached the same result by three independent lines of research: (1) From the cloud observation at 150 stations for about twenty-five years; (2) from the theodolite and nephoscope observations of 1896–97 as given in the Cloud Report; and (3) from the barometric reductions now in operation over the United States and Canada. Furthermore, the theoretical analysis in the Cloud Report makes the solutions by a reversal of gradients entirely improbable, because they depend upon the *existence of warm and cold centers,* which it is well known do not in fact occur. This is easily seen by reference to Chart XV, or to thousands of such abnormal charts in the files of the Weather Bureau. I have already explained, as in fig. 28, MONTHLY WEATHER REVIEW, February, 1903, the process by which the rising air in a cyclone is stripped off by penetrating the eastward drift, involving an interchange of inertia between the local and the general circulations. Also, some further account of the analytic conditions are contained in Paper No. III of this series. It is really very difficult to secure true normal general vectors to use in vector subtraction from the observed vectors in all the subareas surrounding a low center, and in all the cloud strata, and it is no wonder that the work at a single station should be inadequate to such a resolution of forces. Such work has also been done with the prepossession of the old Ferrel meteorology of cyclones, which is very incorrect in many particulars. An inspection of the normal and abnormal isobars of Chart XIV shows that the normal isobars give increased gradients with the height, while the abnormal isobars give diminished gradients with the height, and there is no reason why there should be reversal in either the cyclone or the anticyclone, but simple decrease in the abnormal system until complete disappearance occurs where the general system dominates in the high levels.

It should be noted that while the example of Chart XIV shows that the saddle is directed northward, there are many cases in which the opening of the cyclone is turned to the other quadrants. Thus, the saddle is found frequently in the western quadrant, occasionally in the southern quadrant, and seldom in the eastern quadrant. The opening may often swing around to the northeast, but it rarely points between east and southeast. When the saddle is to the south in the lower strata, it is likely at the same time to be pointing to the north in the upper strata, showing a complete reversal of the structure within the same cyclone as to compass direction, but there is never a gradient reversal from low pressure to high pressure in the cyclone, or from high pressure to low pressure in the anticyclone, so far as known.

CRITICAL REMARKS REGARDING SEVERAL THEORIES OF CYCLONES AND ANTICYCLONES.

From the data in the possession of the Weather Bureau regarding cyclones and anticyclones, it is proper to lay down the following propositions:

(1). Currents of air of different temperatures counterflow in the lower strata of middle latitudes to produce the cyclonic and anticyclonic circulations.

(2). The maximum and the minimum of the abnormal temperatures, that is, the warm and cold areas, are located between and not at the centers of gyration.

(3). The configuration of the local isobars, as distinguished from the isobars that sustain the general circulation, is the same at all levels and of the same type as that at the sea level.

(4). These closed isobars diminish in number with the height until they disappear in the general circulation at moderate elevations, but they do not reverse from low to high pressure or from high to low pressure with the altitude.

(5). Currents of air stream continuously through the cyclone and the anticyclone, so that the circulation involves fresh masses and not a cyclic return of the same masses.

These principles seem to be so thoroughly established that they become criteria for the validity of several proposed methods of the analysis of cyclones and anticyclones, as given in well-known papers on this subject. In case any theory should conflict with the results of observations as given, then the observations must themselves be disproved, or else some substitute found for the theory in question. Several of them were worked out years ago, and had not the advantage of our modern observations, which have materially modified the point of view.

Ferrel's cyclone.—In this cyclone a bounding cylinder is drawn around the circulating mass, excluding it from contact with fresh masses (contra 5); it requires the maximum heat for a warm center cyclone, or the minimum heat for a cold center cyclone to be distributed symmetrically about the axis of gyration (contra 2); the system of isobars undergoes reversal along the axis, as in the warm center cyclone from low pressure at the surface to high pressure above, with corresponding inflow and outflow or reversal of the radial components (contra 3 and 4).

Oberbeck's cyclone.—This cyclone requires a symmetrical distribution of temperature about the center (contra 2); an increase in the vertical velocity in proportion to the height above the surface $w = + cz$, (contra 4), whereas the diminution in number of the closed isobars with the height implies a decrease, $w = - cz$; a concentration of the closed isobars of the inner region near its boundary of separation from the outer region where $w = 0$, (contra 3) since the usual concentration in one quadrant and separation in the opposite quadrant is due to the location of the warm and cold waves and not to a dynamic circulation.

Hann's cyclone.—The theory of cyclones as eddies in as tream having different velocities at the same level requires greater velocity differences in latitude than the general isobars which sustain the eastward drift will admit; since the velocity differences increase with the height, eddy cyclones should especially frequent the upper strata and increase with the altitude (contra 4), but they disappear where they should be strengthening; the temperature distribution as observed can not be continuously maintained on purely hydrodynamic principles.

Hildebrandsson's cyclone.—An eddy cyclone with inflow at the bottom and outflow at the top (contra 1, 2, 3, 4) seems inconsistent with itself, if the gyratory velocity has opposite directions above and below, as claimed to have been indicated in the diagrams and observations of the Blue Hill Observatory.

v. Bjerknes's cyclone.—(Compare Arrhenius's, Kosmische Physik.) This cyclone, deduced from the line integrations, requires warm and cold centers superposed (contra 3), and distributed symmetrically about the axis (contra 2); increase of the closed isobars followed by decrease with the height (contra 4), and varous internal circuits not found in the Weather Bureau observations.

Meinardus describes stream lines based upon the Oberbeck cyclone, and the exposition has to encounter the difficulties mentioned above.

Shaw describes a special case motion quite in conformity with the stream lines in tornadoes, waterspouts, and hurricanes, but not in agreement with such a complex circulation as is found in cyclones of the United States, and typified in Chart XIV.

If these objections continue to be sustained by future observations, it follows that true analytical discussion of the forces in cyclones and anticyclones must avoid such mistaken assumptions as have been laid at the basis of much mathematical meteorology. The actual circulation is really complex in individual cases, and yet it is not difficult to see what in the main the leading principles must be. Further examination of the distribution of the temperature in the higher levels is next in the order of the research.

THE CAUSE OF THE COUNTERCURRENTS IN THE LOWER STRATA.

Ferrel's conception of the general circulation, as derived from a canal theory, where the hot air of the Tropics rises and flows toward the poles in the higher levels, fails to give sufficient account of the persistent southerly winds in the lower strata. Referring to Paper III, of this series, "The problem of the general circulation of the atmosphere of the earth," MONTHLY WEATHER REVIEW, January, 1904, the following remarks suffice. If the temperature of the Tropics is T_1 and of the temperate zone is T_2 in normal conditions, while the heat energy of the Tropics is Q_1 we shall have for the work,

$$W = \frac{Q_1}{T_1} (T_1 - T_2),$$

in average seasonal relations on the rotating earth. Since the solar insolation tends to raise T_1 to $(T_1 + \Delta T_1)$ in the Tropics, and polar radiation changes $- T_2$ to $- (T_2 + \Delta T_2)$, we shall have an increment of work,

$$W + \Delta W = \frac{Q_1 + \Delta Q_1}{T_1 + \Delta T_1} \left[(T_1 + \Delta T_1) - (T_2 + \Delta T_2) \right].$$

Where

$$\Delta Q_1 = (\lambda C_v + \mu C_v^1) \Delta T + A (\lambda R + \mu R_1) T \frac{dv}{v}$$

by formula 112, Cloud Report. The question depends upon the expenditure of the work ΔW, whose purpose is to restore thermal equilibrium as promptly as possible.

The Weather Bureau data show that a system of irregular currents of warm air flow from the Gulf of Mexico upon the United States in the lower strata, and are primary components in the cyclones, where mixing of the currents of air at different temperatures is taking place. These warm streams reach the place of mixture by flowing a short distance across the general high pressure belt, where there is no east and west velocity to complicate the action through an interchange of inertia between the normal masses and the extra currents. In the canal circulation there is on the other hand great opportunity for changes of inertia in both senses, and this the currents tend to avoid unless it is forced upon them. Thus, the warm air of the Tropics in rising first passes to strata of decreasing velocity; then, in moving northward, to strata of increasing velocity; and, finally, in descending in the temperate or polar zones, to strata of diminishing velocity. It is apparent, therefore, that the over-heated masses in the Tropics seek the temperate zone, where they encounter cooler masses, by a short and simple path rather than by a long and very complex path. The tendency for currents of high temperatures to remain individually intact as long as is possible is an additional reason for seeking the cyclonic belt by a direct path in the lower and nearly quiet strata.

Similar reasoning applied to the polar zones gives a sufficient account of the cold western current that enters the cyclone. The warm current on the eastern side underflows the eastward drift, rises into it, and allows the cold current from the northwest to flow beneath. This starts a gyration which is developed as shown by the observations described, and it continues to act as long as the feeding countercurrents of different temperatures endure. The mixture with the eastward drift propels the structure forward, and at the same time breaks up the heated air coming from the Tropics into a succession of irregular cyclones aud anticyclones. The mutual reactions between the constituent parts are so numerous and so subtle that it is not easy to distinguish exactly where one set of forces, as the thermodynamic, ends, and another, as the hydrodynamic, begins. This interplay is, no doubt, responsible for the perplexity that meteorologists have encountered in establishing the correct theory of cyclonic action.

PRECEPTS FOR FORECASTING WITH THE CHARTS ON THE 3500-FOOT AND THE 10,000-FOOT PLANES AS AUXILIARIES.

The chief object in preparing auxiliary pressure and temperature charts on two planes, 3500 feet and 10,000 feet, above the sea level, was to secure three sections through the lower parts of a storm, so that their mutual relations might be studied for the purpose of improving the forecasts. They have proved to be of interest not only in theoretrical meteorology but also in forecasting, as will be more fully set forth in the full report. It has been thought proper by the Chief of the Weather Bureau to give them a practical test at the Washington office during the coming winter before deciding what emphasis shall be placed upon them. It should be clearly understood that they are intended simply to supplement the usual sea-level weather map, as auxillaries of any other kind are employed, such as the pressure-change maps, and the cloud maps now in use in forecasting. It will be necessary only to use a two-syllable code word, as the pressure-temperature word for telegraphing the fractions of the inch of pressure on the two planes, since the integral inches can readily be inferred. The study of these charts has developed many new ideas which it would be impossible to explain in a short paper, and there is no little novelty of thought involved. It will require practise to make good use of all three charts simultaneously, as they are very different from one another, but as there may be some interest in the matter among the forecast officials I have added certain precepts or brief statements, derived as the result of my own experience and studies.

1. *Direction of the storm tracks.*—These follow closely the trend of the isobars on the 10,000-foot plane, reference being made to the 10,000-foot isobars that are closely packed together and are lying well to the south of the center of the cyclones, no regard being paid to the northern curves which are distorted by other influences and represent special features of the circulation.

2. *The velocity of advance.*—This is very closely proportional to the density with which the isobars south of the cyclone are compressed, a strong gradient indicating a quick advance, while a wide gradient with straggling isobars indicates that the velocity of the movement will be slow.

3. *The areas of precipitation.*—To the eastward of the Rocky Mountain Divide these are approximately marked out by the crossing of the isobars on the 3500-foot plane at a good angle beneath those on the 10,000-foot plane. This is especially true of the months from November to April, inclusive. In the summer months, May to October, if the two sets of isobars are crowded together and nearly parallel, there is probability of rainfall in the midst of them. This rule seems to be nearly true for the north Pacific and north Plateau districts throughout the year. The relative positions of the low areas and the high areas is of great significance. On the Pacific coast when a high area is to the south and a low area to the north, these crowded isobars bring precipitation when they run directly from the ocean to the land. If the high area is well to the north, with the low area far inland, and the isobars run from the northwest or north, the tendency to cause precipitation is much diminished. The area of precipitation is on the *western* side of the cyclones of the middle and north Plateau districts, but it shifts to the *eastern* side as the cyclone passes over the Rocky Mountain slope. This makes difficult the forecasting for precipitation in the States of the slope.

If the cyclone is on the southern Plateau or the southern slope and a high area is to the eastward in the Gulf States, the 3500-foot isobars usually open out widely to the south and the region of the underflow marks out the precipitation area very definitely. If the cyclone is located far to the north, over the Missouri Valley, the southern ends of the 3500-foot isobars are usually smooth, unbroken curves, and although there may be a well-marked underflow region, the precipitation area is to be made much smaller than in the preceding case and placed well around on the north side of the cyclone. The precipitation area is apt to be confined to the closed isobars surrounding the center. The difference between these cases lies in the fact that the underflowing current in the southern cyclone is full of moisture from the Gulf of Mexico, but in the northern cyclone it is much drier, even to the extent of being without precipitation. The two types here mentioned will require study and practise for their differentiation, but a working knowledge of the difference is easily acquired. When the cyclone is east of the Mississippi River, its passage eastward or northeastward is closely controlled by the 10,000-foot plane isobars. If the cyclone is on the Atlantic coast, a high-pressure area following it from the southwest should be interpreted as meaning that a rapid clearing will follow over the region of precipitation in the rear of the cyclone. When the cyclone is over the Lake region and a second depression is over the Gulf of St. Lawrence, if the isobars loop northward over the Middle States between the two lows, the area of precipitation is quite general from New England to Minnesota.

4. *Penetration into the higher levels.*—In the winter months, December to March, the heads of the cyclones do not penetrate very much above the two-mile level, but in the summer months, June to September, they are apparently about four miles high. In these seasons, respectively, the power of an individual storm is measured by its penetration into the higher levels and is proportional to it. In the winter the upper strata are cold and drift rapidly eastward, and these two causes deplete the intruding cyclonic head; in the summer the cool strata are much higher and they move slowly, so that an uplifting cyclone finds little to check its development. This principle is seen, also, in the action of cumulo-nimbus clouds in summer. Allowing for relative differences of the seasons, the action of the upper strata upon cyclones is always essentially the same.

While there are several such general principles as these to be acquired by practise, it is yet true that the distinctive weather types are fewer in number and simpler in form on the 3500-foot and the 10,000-foot planes than on the sea level, and apparently their action is much less fluctuating and deceptive. This is a decided advantage in studying the forecast problems, which now suffer by their great complexity on the sea-level plane. In any preliminary study of the new charts there is likely to be some confusion of mind, due to the novelty of the isobaric structure and the intrinsic differences of configuration between the several planes. This mental state will be cleared up by practise, and it will be found that a real addition has been made in the understanding of the prevailing storm action. Further advantage to be derived from an introduction of the isotherms on the upper planes will probably increase the efficiency of this system in other ways.

Chart XII. Isobars on the sea level, the 3500-foot, and the 10,000-foot planes.

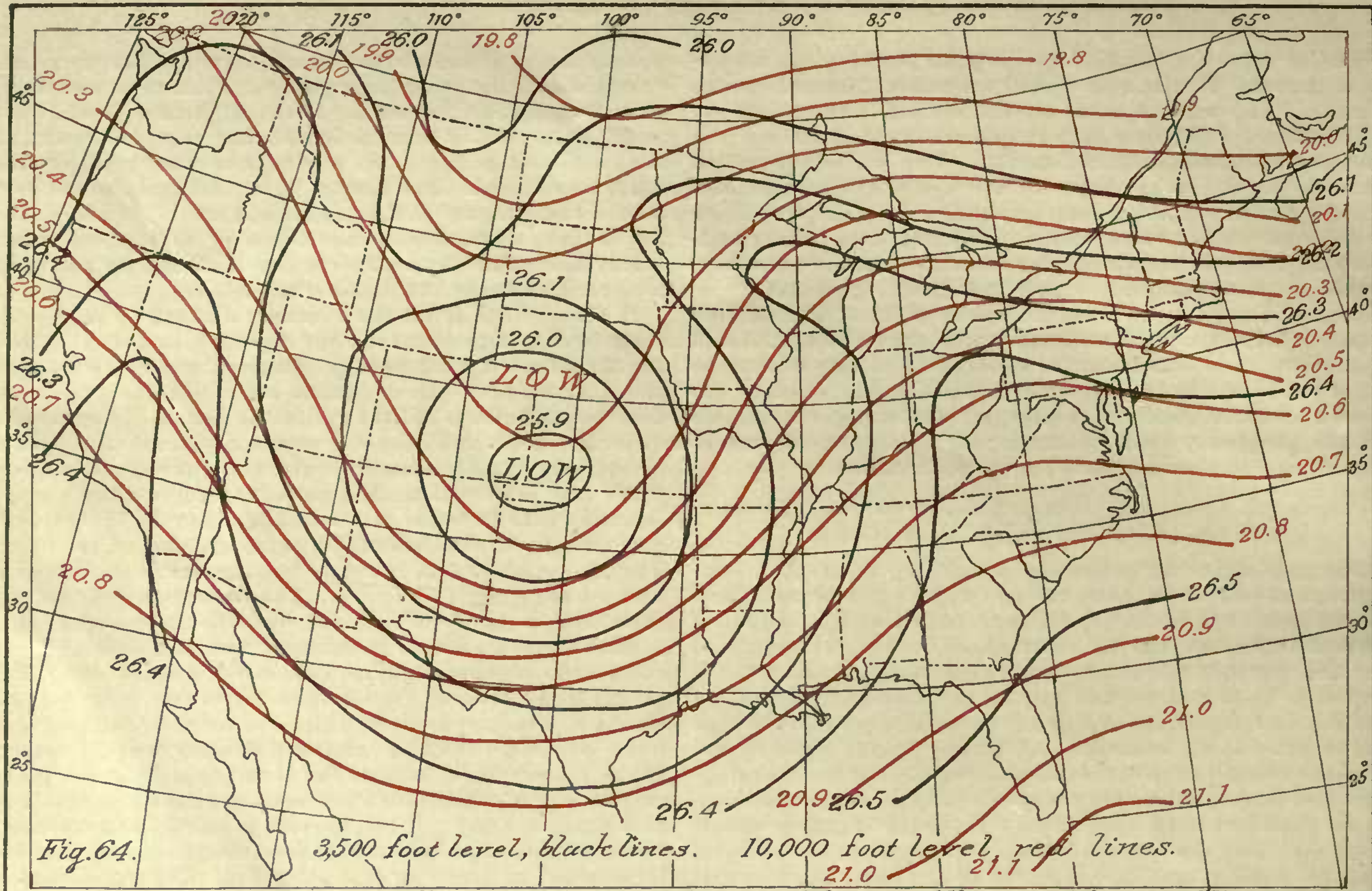


Fig. 64. 3,500 foot level, black lines. 10,000 foot level, red lines.

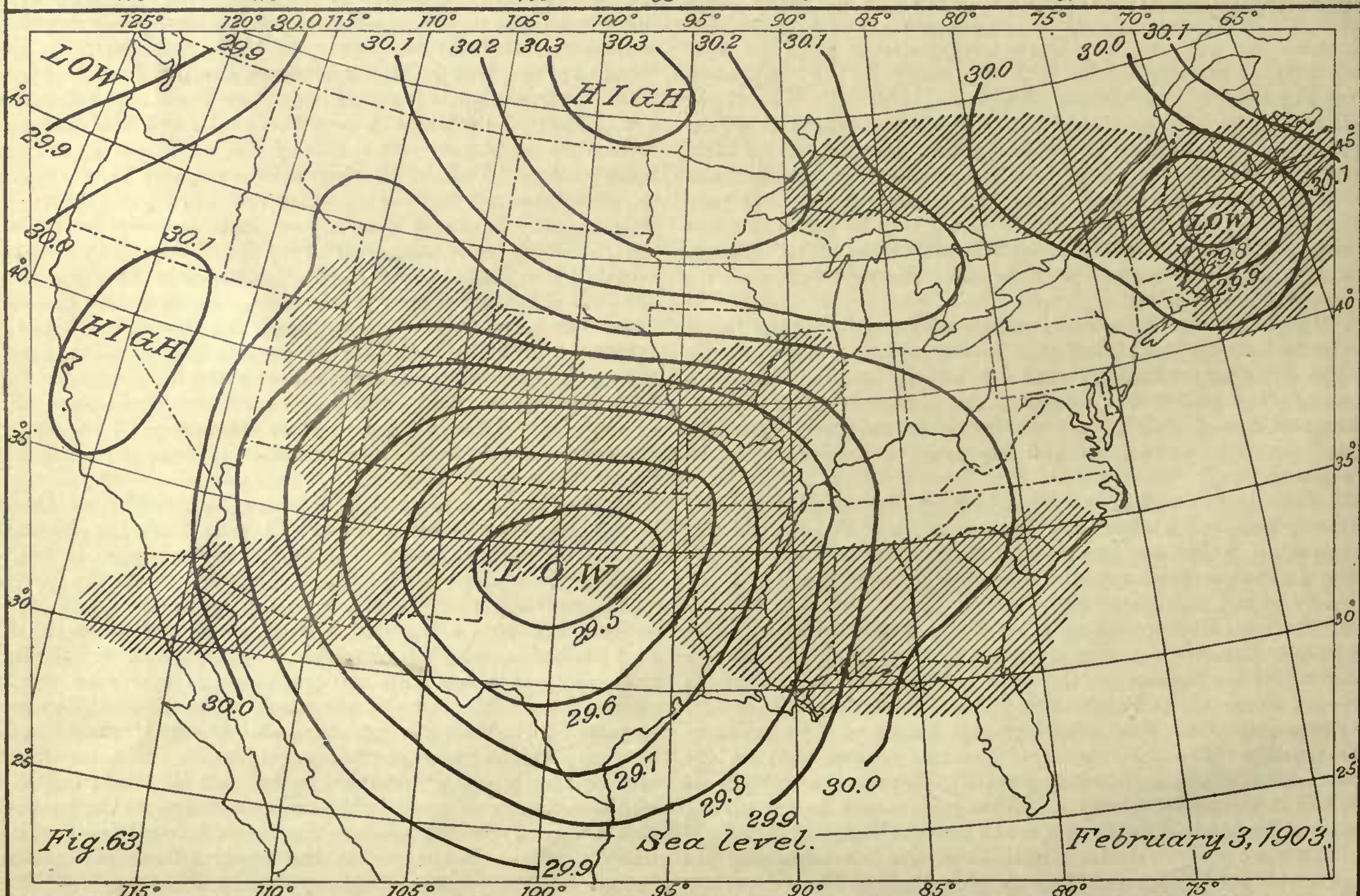


Fig. 63. Sea level. February 3, 1903.

 Chart XIII. Components of pressure on the 3500-foot and the 10,000-foot planes.

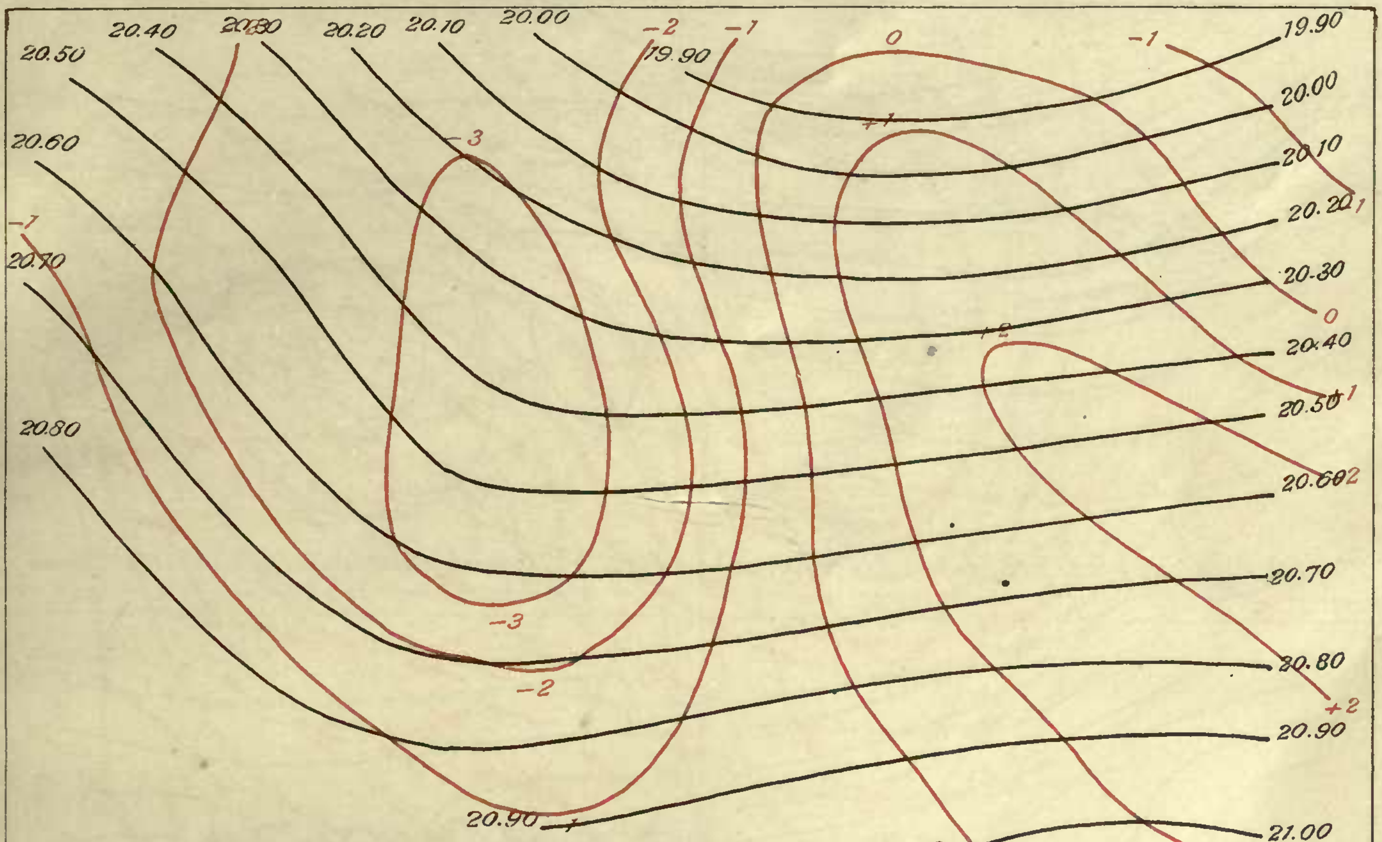


Fig. 66. Isobars. (normal black, abnormal red lines.)

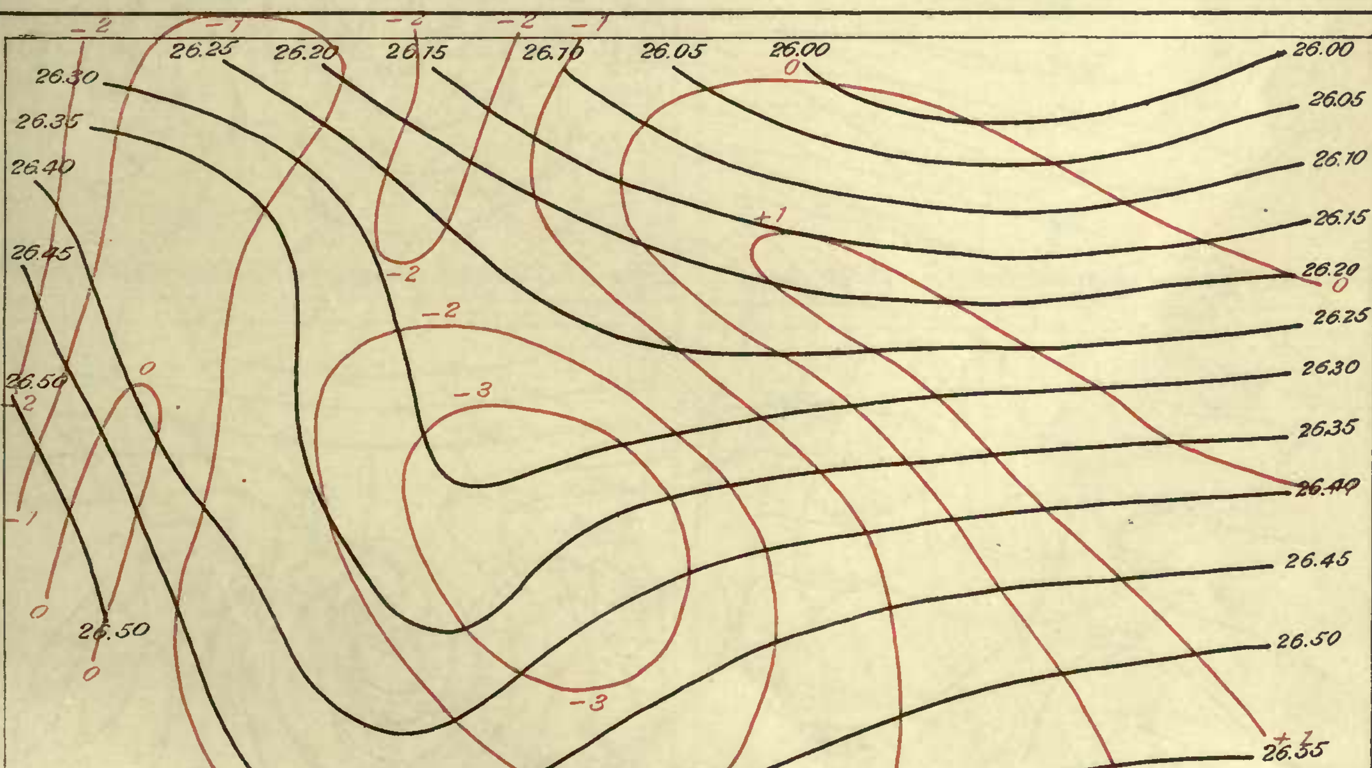


Fig. 65. Isobars. (normal black, abnormal red lines.)

February 3, 1903.

Chart XIV.—Typical distribution of normal and abnormal isobars.

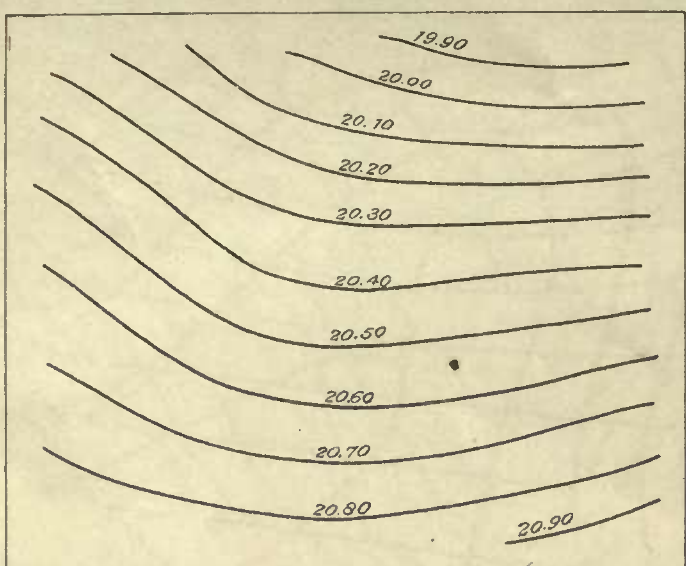


Fig. 69. Typical normal Isobars. (10,000 foot).

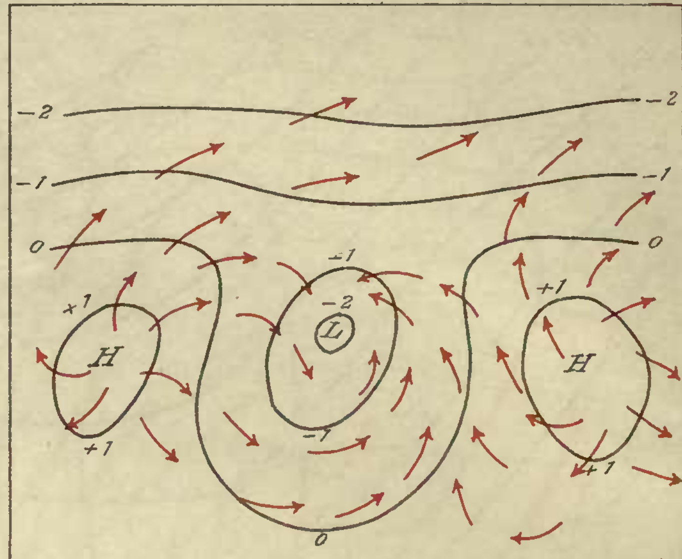


Fig. 72. Typical abnormal Isobars. (10,000 foot).

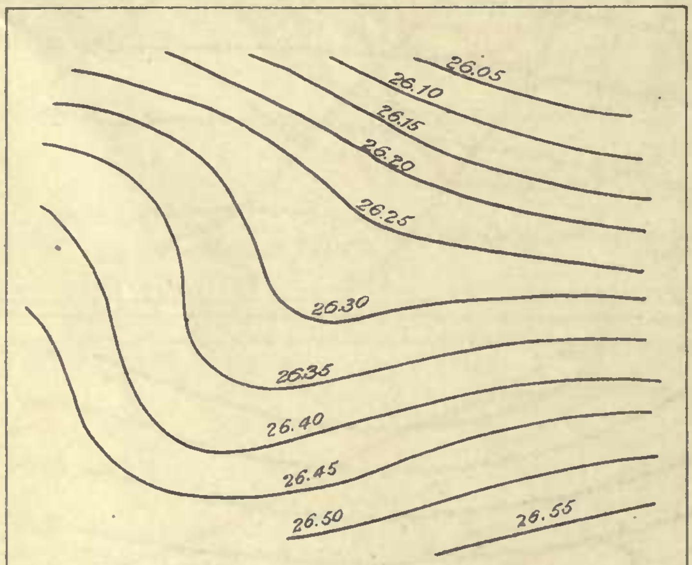


Fig. 68. Typical normal Isobars. (3,500 foot.)

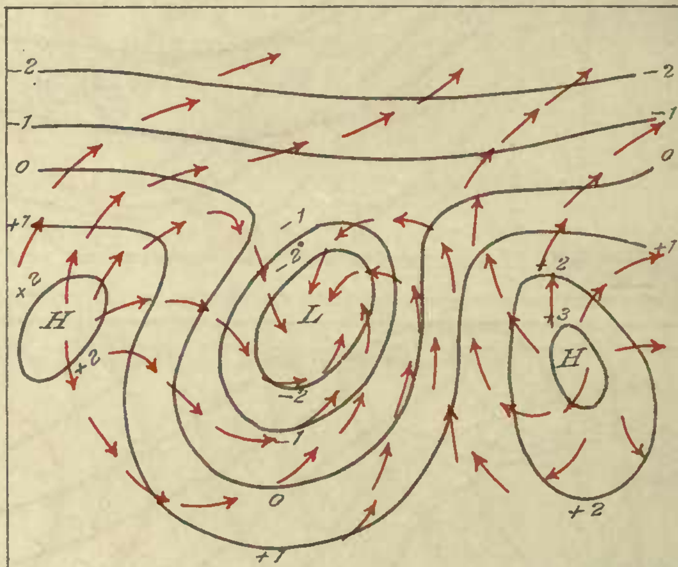


Fig. 71. Typical abnormal Isobars. (3,500 foot.)

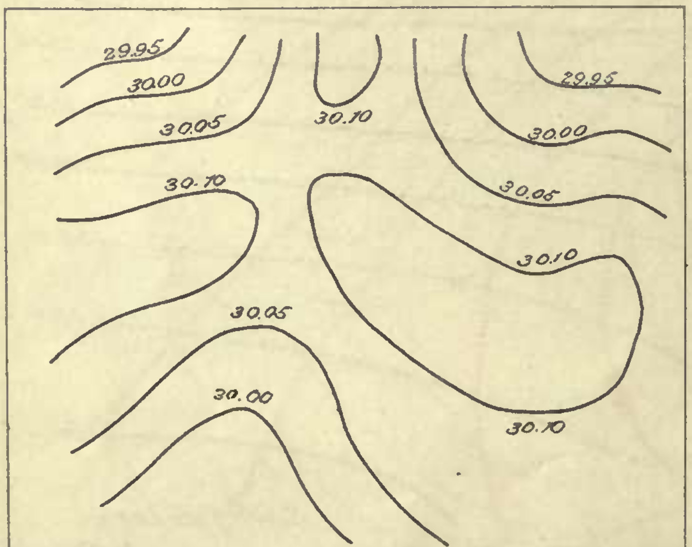


Fig. 67. Typical normal Isobars. (Sea level.)

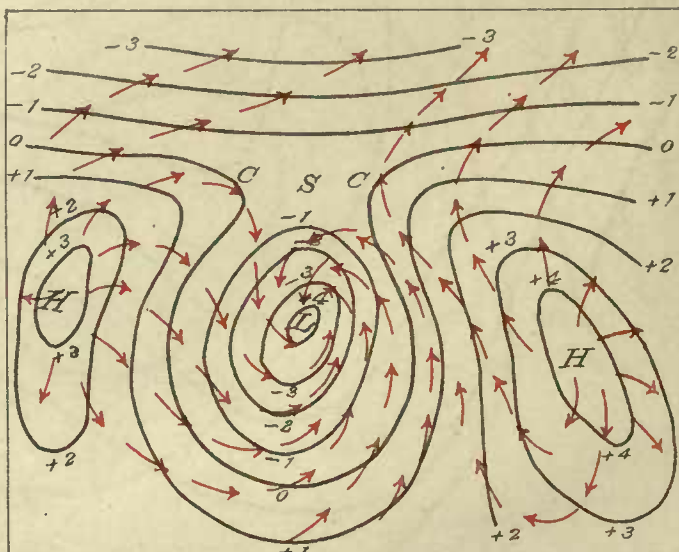


Fig. 70. Typical abnormal Isobars. (Sea level).

Chart XV. Typical normal and abnormal isotherms showing the positions of the maximum excess and deficiency of heat.

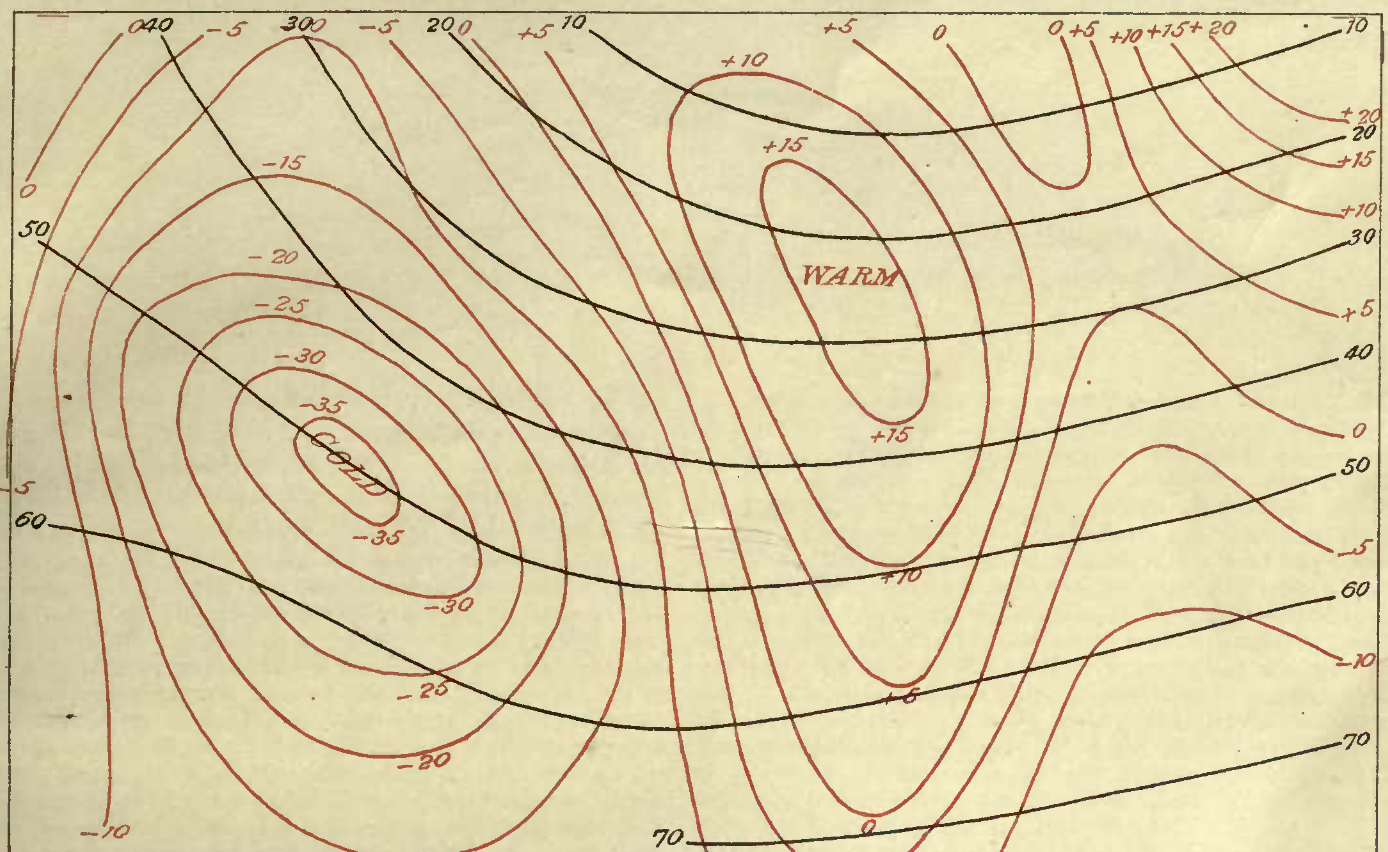


Fig. 74. Temperature Components for February 27, 1903.

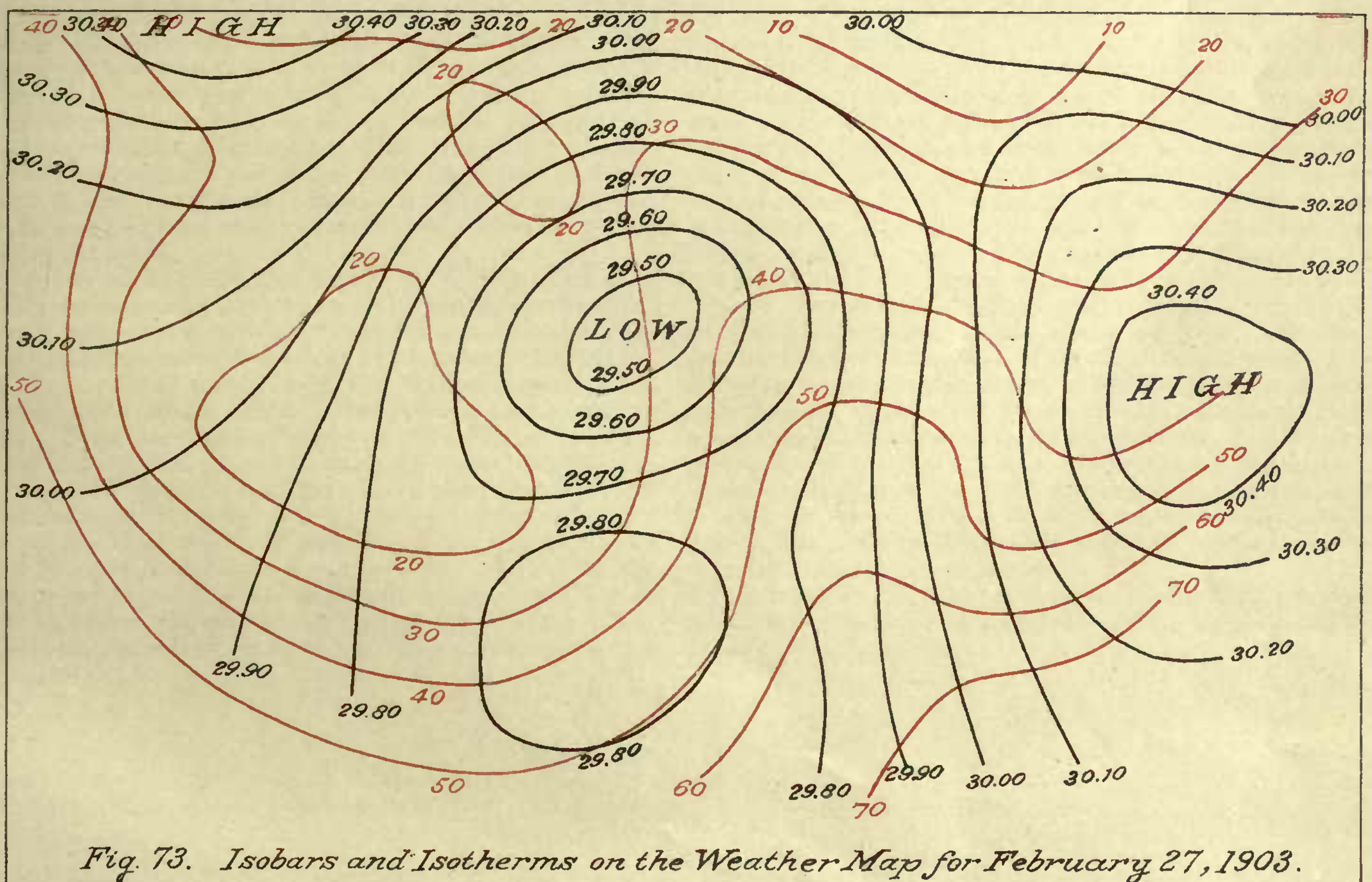


Fig. 73. Isobars and Isotherms on the Weather Map for February 27, 1903.

LIBRARY
OF THE
UNIVERSITY
OF
CALIFORNIA

VII.—THE AVERAGE MONTHLY VECTORS OF THE GENERAL CIRCULATION IN THE UNITED STATES.

In Table 9, page 144, Annual Report of the Chief of the Weather Bureau, 1898–1899, may be found the data resulting from the nephoscope observations taken in the international cloud year, 1896–1897, which were made to determine the general motions of the atmosphere over the United States. In Table 33, page 409, of the same volume, is given a summary of the resulting general velocities as annual normals. It remains to compute the mean monthly normal vectors of the circulation, and it has been done by the methods used in computing similar vectors for the West Indies, so that but few preliminary remarks are needed in this connection. The method now in use in the Weather Bureau of determining the monthly direction of the wind at a station is really inadequate to the requirements of modern science, which demands an accurate knowledge of the azimuth direction and velocity of the wind. The method referred to consists in counting the number of times the wind was reported on each of the eight cardinal points, N., NE., E., etc., and assigning as the monthly direction that which has the plurality of numbers. This gives no true resultant direction and takes no account of the velocity of the wind prevailing at each observation. A second method of reducing wind observations, which is somewhat more accurate than the former, consists in assuming an equal velocity for each wind and combining the frequency numbers by using Lambert's formula or its equivalent. This system gives a true resultant direction for winds of uniform velocity, but where the winds are variable in force, as well as in direction, this is also insufficient. Many examples of inaccurate resultants can be given when the individual velocities are not constantly the same.

The vectors of Table 16, and figs. 77 to 88, Charts XI, XII, and XIII, "Average monthly vectors of the general circulation," have been computed accurately by resolving each vector V_1 φ, as observed, into its north to south and west to east components, taking the algebraic sum of each, and thence computing the mean component for the series, in this case for each month of the year. Then the resultant vectors in velocity and azimuth were constructed, and appear in Table 16 under the columns V, φ. Since the resultant vectors in the lower cloud level and at the surface are very small, I have also computed the mean motion of the wind for each month without regard to the azimuth direction, and this is given under V_1. In the middle and the upper cloud strata the azimuth directions are not so variable as nearer the surface, and hence, there is less difference between the values of V_1 and V. The resultant vectors V, φ have been plotted in two arrangements, the first giving the vectors of the month for each cloud system terminating on the same vertical lines, which permits a ready inspection of the relative motion in the different levels for each month of the year. The second arrangement gives the vectors for June ending on one vertical line, while those for the other months follow in a broken line, which shows at a glance the trend of the circulation throughout the year in the several cloud groups. It has been convenient to divide the clouds into three groups, (1) the lower clouds (stratus, cumulus, strato-cumulus), (2) the middle clouds (alto-stratus, alto-cumulus), and (3) the upper clouds (cirro-cumulus, cirro-stratus, cirrus), which do not differ greatly among themselves in velocity. The average height of group (1), lower clouds, is 2000 meters; of group (2), middle clouds, 5000 meters, and of group (3), upper clouds, 9000 meters, as determined by the theodolite observations at Washington, in 1896–1897.

We make the following remarks on the vectors of Charts XI to XIII. The northern group of stations, St. Paul, Detroit, Cleveland, Buffalo, Louisville, Blue Hill, Washington, Waynesville, and Ocean City, all lie in the strong eastward drift to the north of the high pressure belt of the general circulation; Kansas City, Abilene, and Vicksburg, lie in the midst of this belt, while Key West is on the southern border of it and has some of the characteristics of the West Indian group of stations. The northern stations in the upper levels have strong eastward components, and in the lower levels a turbulent circulation with small resultant vectors. Louisville seems to have something like a personal equation, which has magnified the vectors a little above the apparent average that the entire set would suggest, while Cleveland, on the other hand, seems to have a diminished set of vectors. It is not possible to show from the observations what change, if any, ought to be introduced by means of a modifying factor. Besides the relative lengths of the vectors in the different levels it is interesting to note the north and south components at the several stations. Thus, at St. Paul and also at Kansas City, there is a northward component in the cirrus levels; this component prevails at all levels at Abilene. At Vicksburg the vectors are generally small, and they are westward during certain months in the lower strata. At Key West the westward component prevails in the lower levels, but the eastward in the cirrus level, as in the Cuban stations generally.

It is desirable to extend such vector computations to various portions of the earth in order to obtain the data needed in dynamic meteorology.

TABLE 16.—*Average monthly vectors of the general circulation in the United States at four levels.*

1. ST. PAUL, MINN.

1896–97.	Wind.			St., Cu., S. Cu.			A. S., A. Cu.			Ci. Cu., Ci. S., Ci.		
	Velocity in meters per second.											
	V_1	V	φ	V_1	V	φ	V_1	V	φ	V_1	V	φ
			°			°			°			°
January	3.9	1.7	48	27.6	16.8	80	52.5	38.0	90	72.9	36.9	95
February	4.1	1.5	37	27.4	14.4	78	54.5	29.5	91	72.9	16.2	117
March	4.1	1.1	27	26.4	12.2	81	54.0	20.5	97	72.0	10.8	158
April	4.1	0.5	7	25.2	10.0	87	52.5	14.5	109	66.6	14.5	150
May	4.0	0.0	180	23.6	8.2	92	50.0	12.0	122	71.1	23.4	128
June	3.7	0.5	174	22.4	7.6	102	48.5	12.5	122	55.8	39.6	113
July	3.6	0.8	163	21.8	8.0	103	46.0	18.5	113	54.0	48.6	106
August	3.6	1.0	142	21.4	9.0	102	45.0	26.0	106	52.2	54.9	102
September	3.5	1.2	126	22.0	11.4	96	45.0	35.0	100	53.1	58.5	98
October	3.5	1.4	107	22.8	14.2	91	46.0	40.0	96	55.8	57.6	93
November	3.6	1.5	91	24.0	17.2	86	49.0	42.0	92	63.0	54.0	91
December	3.7	1.4	69	25.6	18.0	82	50.5	41.5	90	70.2	46.8	90

2. KANSAS CITY, MO.

January	4.1	1.1	79	22.0	12.0	74	27.0	23.5	76	40.5	36.9	95
February	4.1	0.9	86	21.6	11.0	75	27.5	23.5	75	42.3	35.1	94
March	3.9	0.5	117	19.8	9.6	79	25.0	20.5	77	36.0	28.8	94
April	3.6	0.6	168	17.6	8.0	83	20.0	16.0	82	27.0	24.6	104
May	3.4	0.9	193	15.6	6.2	76	15.0	12.0	92	18.9	18.9	117
June	3.3	1.1	207	14.0	5.2	91	12.0	9.0	111	15.3	18.0	133
July	3.1	1.2	211	14.0	5.4	90	11.0	7.5	121	13.5	18.0	139
August	3.1	1.1	212	14.0	5.6	90	11.0	6.5	122	15.3	18.0	142
September	3.3	0.9	206	15.0	6.6	85	13.0	7.5	97	18.0	18.0	136
October	3.4	0.5	180	16.0	8.0	83	15.5	10.0	82	24.3	18.0	125
November	3.6	0.5	120	18.0	9.8	80	20.0	15.5	76	30.6	22.5	109
December	3.8	0.9	85	20.0	11.0	75	23.5	20.0	76	36.0	29.7	98

3. ABILENE, TEX.

January	4.5	1.2	128	14.6	12.0	116	24.5	23.5	134	41.4	36.0	125
February	4.8	1.1	130	15.2	5.6	120	25.0	25.0	132	44.1	42.3	126
March	4.8	1.1	164	14.2	5.4	147	24.0	22.0	131	41.4	39.6	125
April	4.5	1.5	182	12.6	5.4	165	20.0	18.0	128	34.2	32.4	124
May	4.3	2.1	192	10.8	5.2	180	15.0	13.0	129	25.2	20.7	129
June	4.0	2.6	196	9.0	4.8	193	11.0	8.0	132	18.0	9.0	135
July	3.9	2.7	197	8.0	4.0	206	10.0	3.0	129	13.5	2.7	180
August	3.7	2.5	195	8.0	3.6	213	10.0	1.5	135	10.8	2.7	161
September	3.7	2.0	193	8.8	4.0	203	11.0	3.5	135	14.4	3.6	166
October	3.9	1.5	173	10.0	4.2	182	15.0	8.5	126	18.9	16.2	124
November	4.1	1.2	147	11.4	5.2	159	18.5	15.0	132	27.0	19.8	118
December	4.4	1.3	128	13.4	5.8	137	22.5	21.0	133	34.2	28.8	118

4. VICKSBURG, MISS.

January	2.9	0.5	270	8.2	3.0	98	12.0	13.5	96	18.9	25.2	94
February	3.2	0.2	248	9.0	4.4	101	15.0	14.0	98	25.2	26.1	87
March	3.5	0.4	113	9.0	4.8	112	15.0	12.5	97	27.0	25.2	85
April	3.3	0.9	90	8.0	3.2	120	14.0	11.0	92	25.2	18.0	90
May	3.1	1.2	84	6.8	2.0	126	10.5	8.5	90	20.1	12.6	94
June	2.8	1.3	78	5.6	0.4	90	8.0	5.0	84	16.2	7.2	114
July	2.6	1.1	70	4.8	0.4	0	6.0	2.5	78	12.6	3.6	180
August	2.3	0.8	63	4.4	0.2	313	5.0	1.0	60	9.9	4.5	206
September	2.3	0.3	26	4.4	0.4	245	5.5	1.0	130	9.0	1.8	180
October	2.4	0.4	276	4.8	1.0	201	6.5	3.0	149	9.0	4.5	90
November	2.6	0.8	267	5.6	2.0	173	9.0	6.0	121	10.8	11.7	72
December	2.7	0.8	263	6.2	2.2	146	11.0	8.5	104	15.3	16.2	71

TABLE 16.—*Average monthly vectors of the general circulation, etc.*—Continued.

5. LOUISVILLE, KY.

1896–97.	Velocity in meters per second.											
	Wind.			St., Cu., St. Cu.			A. S., A. Cu.			Ci. Cu., C. S., Ci.		
	V_1	V	φ	V_1	V	φ	V_1	V	φ	V_1	V	φ
			°			°			°			°
January	4.0	2.4	130	25.4	23.6	87	49.0	43.5	105	63.0	59.4	90
February	4.1	2.2	128	25.2	22.6	84	48.5	40.5	104	60.3	54.0	94
March	4.1	1 9	131	24.0	20.6	82	42.5	33.0	97	52.2	45.0	96
April	4.0	1.4	132	22.0	18.0	79	35.0	26.0	88	41.4	37.8	87
May	3.7	0.9	124	18.0	15.0	80	27.0	20.5	78	34.2	30.6	76
June	3.5	0.6	108	15.4	12.6	81	24.0	18.5	73	28.8	28.8	71
July	3.2	0.6	104	14.4	11.6	86	24.0	19.5	77	29.7	27.0	67
August	3.0	0.6	104	14.4	12.0	93	25.5	21.5	81	34.2	30.6	75
September	2.9	0.8	114	15.6	13.8	100	30.0	26.0	90	39.2	36.0	84
October	3.1	0.9	141	17.6	16.8	101	35.5	31.5	97	52.2	43.2	95
November	3.3	1.7	128	20.4	20.4	100	41.5	38.0	103	58.5	49.5	104
December	3.6	2.0	132	23.2	22.0	95	46.5	43.0	105	61.2	55.8	103

6. DETROIT, MICH.

January	5.0	3.1	117	40.6	33.2	93	41.5	30.5	100	55.8	52.2	91
February	5.1	2.8	111	38.8	31.2	92	42.0	35.5	98	54.0	52.2	91
March	5.0	2.4	105	35.0	28.0	90	40.0	33.5	97	50.4	48.6	88
April	4.6	2.0	101	29.2	23.2	84	37.5	30.0	94	45.9	45.0	83
May	4.1	1.5	98	23.4	19.0	80	34.0	26.0	91	43.2	41.4	74
June	3.6	0.9	96	20.2	16.2	75	31.5	24.0	88	42.3	40.5	63
July	3.4	0.8	103	19.0	14.4	75	31.5	22.5	86	42.3	39.6	61
August	3.4	0.9	119	19.6	13.6	81	32.0	21.0	87	43.2	40.5	63
September	3.6	1.5	122	22.0	13.8	90	34.5	23.0	92	46.8	40.5	70
October	4.1	2.0	122	26.8	15.8	97	37.0	25.0	95	54.0	44.1	80
November	4.4	2.6	121	34.0	20.4	96	40.0	29.0	99	56.7	48.9	88
December	4.8	3.0	119	40.0	27.6	95	42.0	32.0	99	58.5	49.5	89

7. CLEVELAND, OHIO.

January	5.3	3.3	106	22.6	17.4	103	22.5	22.5	119	41.4	33.3	93
February	5.4	3.2	103	22.4	17.0	104	23.5	21.0	116	39.6	34.2	91
March	5.1	2.8	99	21.0	15.0	102	22.5	17.5	106	36.0	32.4	90
April	4.6	2.4	90	17.6	12.0	101	20.0	15.0	98	27.0	27.0	86
May	4.1	2.0	79	14.0	9.4	94	18.5	13.5	91	23.4	20.7	82
June	3.7	1.8	70	12.2	8.0	86	16.0	12.5	85	18.9	16.2	71
July	3.6	1.5	61	11.4	7.8	72	15.5	13.0	83	18.0	14.4	68
August	3.7	1.4	60	12.0	8.2	63	16.0	14.5	80	18.0	14.4	68
September	4.2	1.5	72	14.0	9.0	63	17.5	16.0	82	19.8	17.1	75
October	4.6	1.8	90	16.4	10.2	70	19.5	18.5	86	25.2	19.8	76
November	5.0	2.3	102	18.4	12.0	82	21.0	19.5	90	32.4	25.2	83
December	5.2	2.8	109	20.4	14.4	93	23.0	20.0	94	37.8	30.7	85

8. BUFFALO, N. Y.

January	5.7	4.2	84	27.4	17.0	86	59.0	44.5	89	51.3	51.3	75
February	5.7	4.1	87	26.4	16.4	84	58.5	43.0	88	53.2	47.7	72
March	5.7	3.7	93	24.0	15.0	87	54.0	36.0	88	36.0	38.7	73
April	5.4	3.2	104	20.6	12.8	91	45.0	30.5	89	33.0	32.4	80
May	5.3	2.9	112	16.8	10.2	96	31.5	23.5	92	33.3	29.7	90
June	5.0	2.6	121	14.0	9.6	102	25.0	18.0	97	35.1	29.7	98
July	4.9	2.3	126	12.0	9.8	104	20.0	15.5	105	36.9	30.6	102
August	4.7	2.3	127	13.2	10.0	105	19.5	16.5	108	41.4	33.3	104
September	4.8	2.3	126	17.0	10.8	102	17.0	19.5	104	45.9	38.7	100
October	5.0	2.2	119	22.0	12.2	96	27.0	23.5	96	52.2	45.0	95
November	5.1	2.4	110	25.4	14.4	91	36.0	30.5	94	54.0	51.3	90
December	5.4	3.2	105	27.2	16.4	87	48.0	39.0	91	55.8	52.2	84

TABLE 16.—*Average monthly vectors of the general circulation, etc.*—Continued.

9. BLUE HILL, MASS.

1896–97.	Velocity in meters per second.											
	Wind.			St., Cu., S. Cu.			A. S., A. Cu.			Ci. Cu., Ci. S., Ci.		
	V_1	V	φ	V_1	V	φ	V_1	V	φ	V_1	V	φ
			°			°			°			°
January	7.6	4.6	68	24.2	12.4	73	38.5	32.0	97	54.0	46.8	87
February	7.2	4.5	62	23.6	12.2	75	35.5	30.5	92	49.5	45.0	83
March	6.8	3.7	61	22.0	11.4	76	32.0	26.5	89	41.4	38.7	80
April	6.4	2.6	88	20.4	10.6	80	28.0	22.0	87	35.1	32.4	83
May	6.2	2.4	116	19.2	9.6	84	24.0	19.0	85	28.8	26.1	87
June	6.1	2.3	126	18.0	8.4	89	22.0	17.5	88	27.0	23.4	94
July	6.2	2.3	129	17.6	8.4	101	19.0	16.5	93	27.0	22.5	98
August	6.3	2.3	130	17.6	9.4	117	20.0	18.5	103	27.9	22.5	99
September	6.8	2.3	125	18.6	10.2	124	25.0	22.0	109	31.5	27.0	101
October	7.0	2.5	114	20.2	10.0	120	31.0	27.0	113	38.7	33.0	99
November	7.3	2.8	98	22.0	9.4	90	36.0	32.0	111	45.0	40.5	96
December	7.6	3.6	80	23.4	10.8	74	38.5	32.5	106	52.2	46.8	93

10. WASHINGTON, D. C.

January	4.1	2.0	34	14.0	8.6	101	27.5	25.0	90	47.7	45.9	91
February	3.9	1.7	23	14.0	8.4	106	28.5	24.0	97	50.4	45.9	92
March	3.6	1.4	39	13.4	8.0	108	26.5	22.5	101	45.0	43.2	92
April	3.2	1.0	49	12.0	7.4	111	22.5	19.5	104	36.0	35.1	91
May	2.8	0.6	73	11.0	6.0	111	18.5	16.0	107	27.0	26.1	90
June	2.5	0.5	110	10.0	5.2	104	15.0	12.0	100	23.4	19.8	90
July	2.3	0.5	117	8.8	4.6	95	13.5	10.5	95	19.8	17.1	90
August	2.3	0.5	117	8.2	4.8	81	13.5	10.0	90	19.8	18.0	88
September	2.5	0.5	90	8.8	5.8	72	15.0	11.5	85	21.6	20.7	87
October	2.9	0.7	65	10.2	7.4	73	18.0	15.0	81	27.0	24.3	87
November	3.2	1.2	50	12.2	8.4	76	21.0	14.5	79	31.5	31.5	90
December	3.6	1.7	40	14.0	8.0	85	25.0	17.5	82	37.8	38.7	90

11. WAYNESVILLE, N. C. 12. OCEAN CITY, MD.

January	2.9	1.6	62	15.8	11.6	91	36.0	35.0	108	53.1	51.3	94
February	2.7	1.4	59	15.2	11.2	92	35.5	35.0	107	54.0	51.3	94
March	2.3	0.9	57	13.2	10.0	90	34.0	31.5	109	45.0	45.0	92
April	1.9	0.5	56	12.6	8.0	88	30.0	26.5	108	36.0	35.1	91
May	1.6	0.1	107	8.4	6.0	80	25.0	19.0	103	27.0	25.2	91
June	1.4	0.3	180	6.8	4.8	70	18.5	15.0	95	18.0	14.4	90
July	1.3	0.5	180	6.0	4.6	63	13.0	10.0	91	10.8	9.0	84
August	1.3	0.4	167	6.0	4.8	68	11.0	10.5	90	9.0	9.0	84
September	1.4	0.4	120	8.0	6.4	67	14.0	14.0	94	11.7	14.4	86
October	2.0	0.6	78	10.4	8.0	85	20.0	20.5	98	18.9	26.1	88
November	2.6	1.1	68	13.6	9.8	87	26.5	27.5	102	28.8	36.0	90
December	2.9	1.5	63	15.6	11.2	91	34.0	32.0	105	39.6	45.0	92

13. KEY WEST, FLA.

January	4.6	2.3	318	14.4	2.4	215	20.0	8.0	97	28.8	26.1	90
February	4.5	1.8	294	15.2	4.6	193	20.0	9.5	106	27.9	24.3	91
March	4.3	1.9	265	14.8	6.4	197	17.0	7.5	106	25.2	19.8	86
April	4.1	2.4	251	13.6	8.0	210	13.5	4.0	104	18.0	14.4	69
May	4.2	2.7	249	12.0	9.0	221	10.0	0.5	135	11.7	10.8	49
June	4.1	2.9	248	10.4	8.8	228	8.5	2.0	270	17.1	7.2	7
July	4.0	2.9	253	9.4	11.0	244	7.5	5.0	259	15.3	5.4	321
August	3.8	2.9	261	8.4	10.0	254	7.5	5.5	255	15.3	4.5	308
September	3.8	2.8	274	8.4	8.0	262	8.5	7.0	249	9.0	1.8	227
October	4.1	2.7	289	9.2	4.8	267	10.5	5.5	246	12.6	7.2	120
November	4.5	2.7	306	10.2	2.4	260	14.5	4.0	235	18.0	15.3	107
December	4.6	2.7	318	12.2	1.8	229	17.0	3.0	211	23.4	19.8	100

Chart XI. Average monthly vectors of the general circulation.

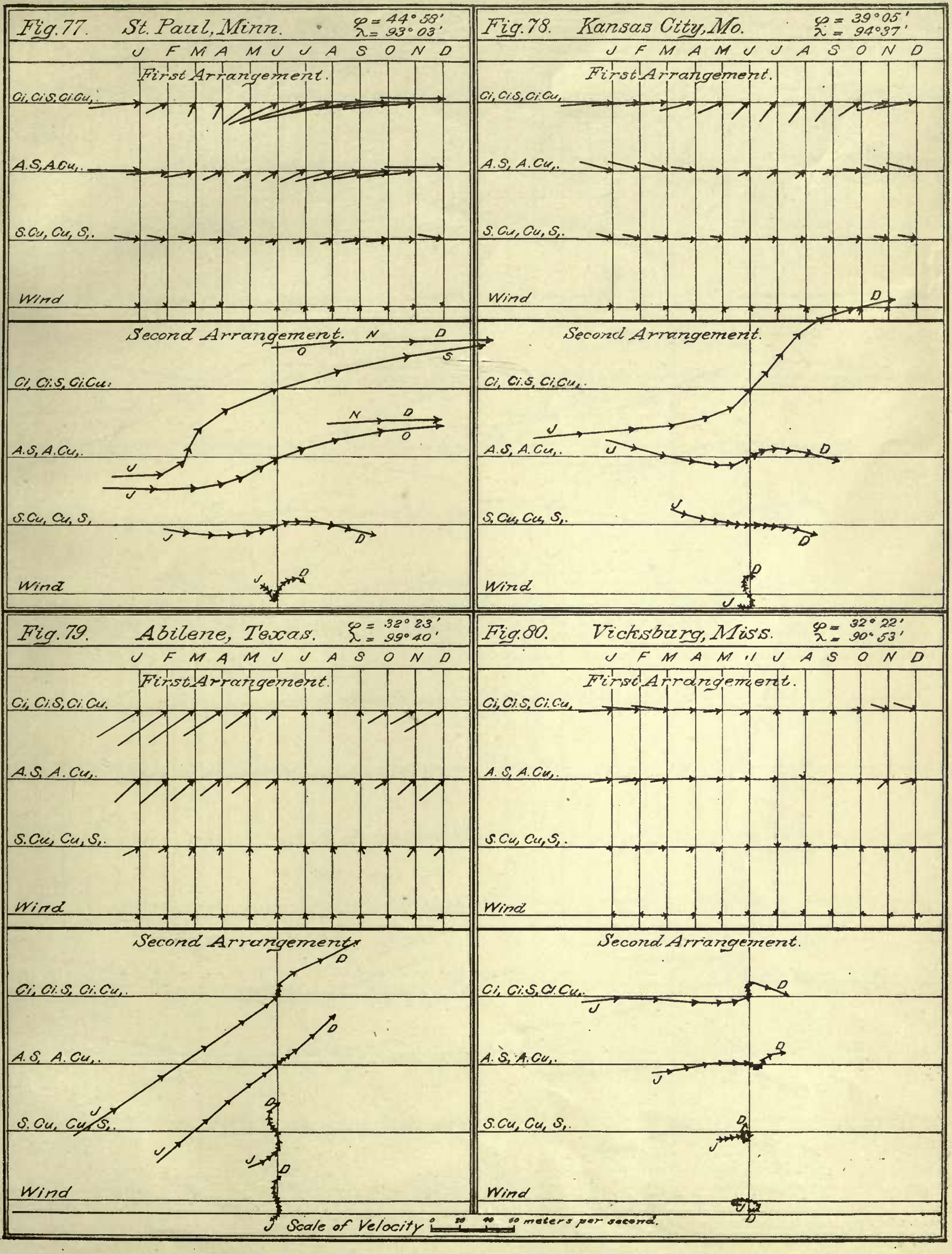


Chart XII. Average monthly vectors of the general circulation.

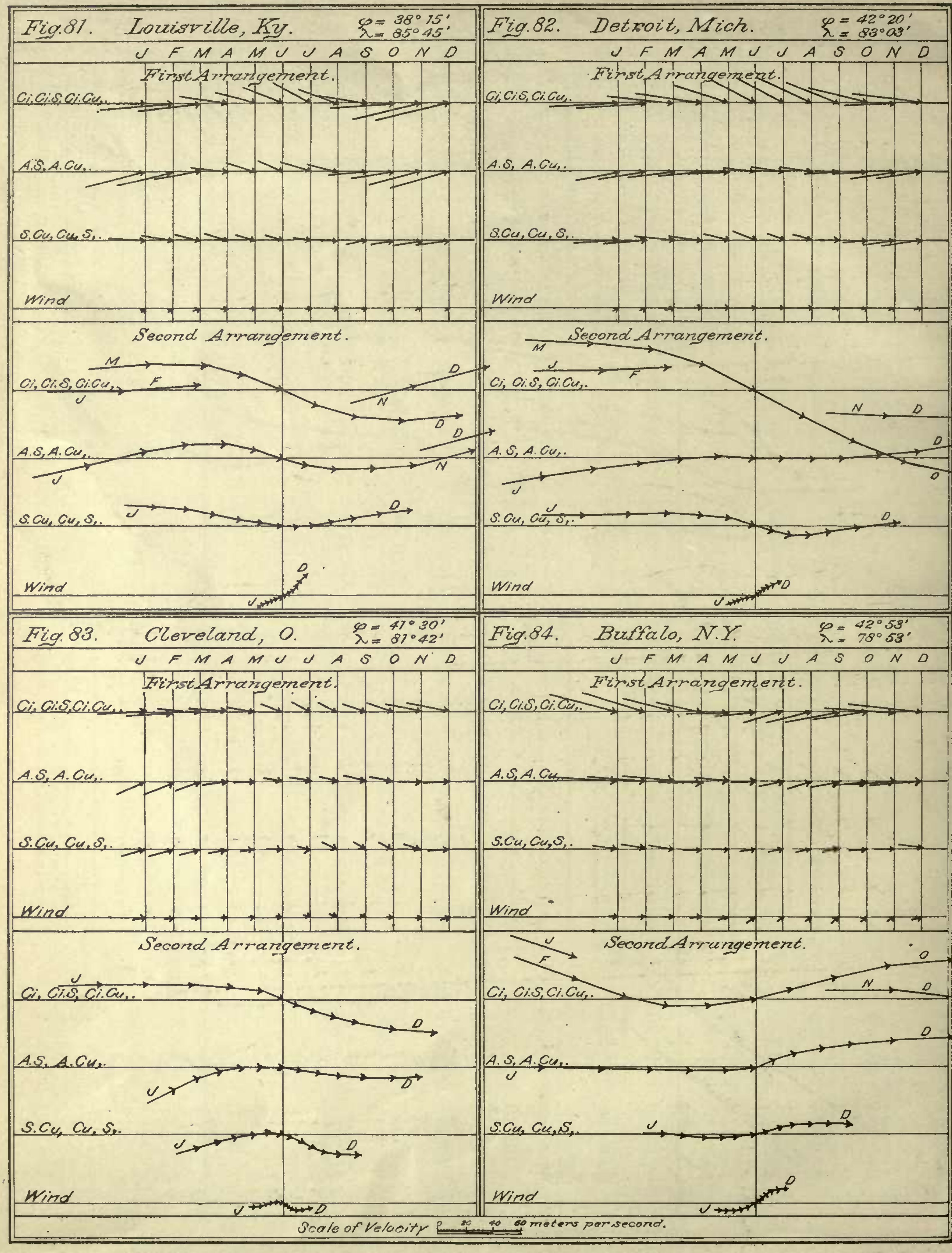
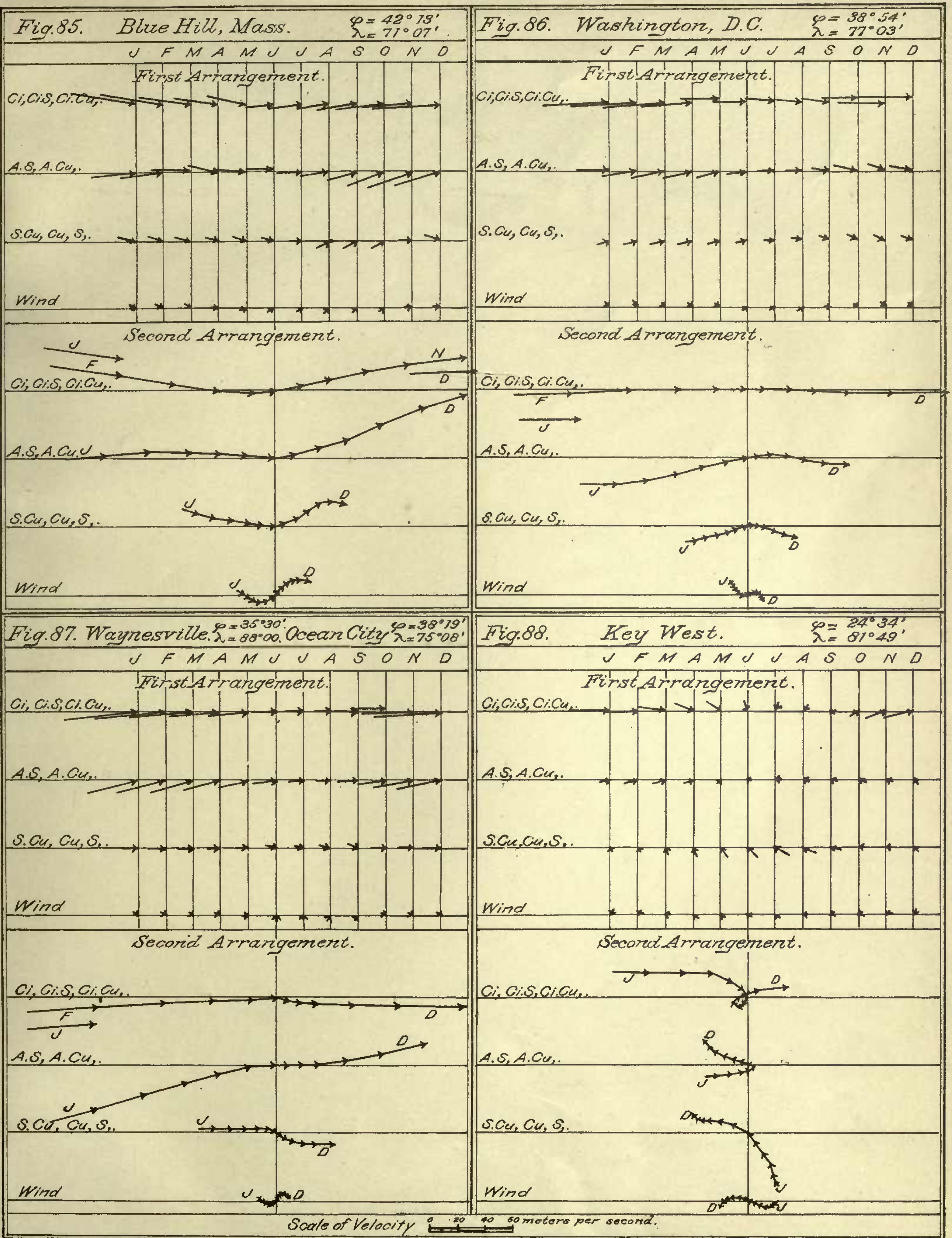


Chart XIII. Average monthly vectors of the general circulation.



LIBRARY
OF THE
UNIVERSITY
OF
CALIFORNIA

RETURN TO → **CIRCULATION DEPARTMENT** 23110
202 Main Library

LOAN PERIOD 1 **HOME USE**	2	3
4	5	6

ALL BOOKS MAY BE RECALLED AFTER 7 DAYS
Renewals and Recharges may be made 4 days prior to the due date.
Books may be Renewed by calling 642-3405.

DUE AS STAMPED BELOW

LIBRARY USE ONLY
AUG 18 1989
CIRCULATION DEPT.

RECEIVED
AUG 18 1989
CIRCULATION DEPT.

FORM NO. DD6
UNIVERSITY OF CALIFORNIA, BERKELEY
BERKELEY, CA 94720

[80m-6,'11]

U.C. BERKELEY LIBRARIES
C020873260
128549
QC931
B5